NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Problem of the Week Teacher Packet

I Get a Kick Out of Soccer

As I watched our high school soccer team practice, I heard the coach instruct the team to do laps and then hit the showers. I got to wondering how many times they would need to run around the field to complete at least one mile.

I walked over to the coach and asked for the dimensions of the field. He said that our soccer field is 75 yards wide and 115 yards long.

How many laps would a team member need to complete to run more than a mile?

Extra: If each of the 22 team members runs four laps before practice and two laps after practice, how many miles will be run by the team each day?

Answer Check

After students submit their solution, they can choose to "check" their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually get the answer) we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

After completing 5 laps a player would have run more than a mile.

If your answer **doesn't** match ours,

- did you know there are 1760 yards in a mile?
- did you remember that the field is a rectangle?
- did you realize that running a lap means running around the four sides of the field?
- did you check your arithmetic?
- did you get 4.64 or a number like it? Notice that the problem asks how many laps they would complete.

If any of those ideas help you, you might revise your answer, and then leave a comment that tells us what you did. If you're still stuck, leave a comment that tells us where you think you need help.

If your answer **does** match ours,

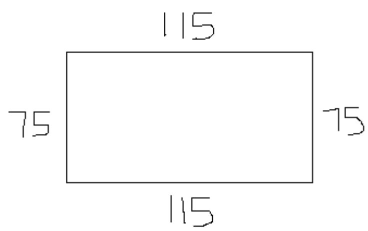
- did you try the Extra question?
- is your explanation clear and complete?
- did you verify your answers with another method?
- what hints would you give another student trying to solve this problem?
- did you have any "Aha!" moments or notice any patterns? Describe them.

Revise your work if you have any ideas to add. Otherwise leave us a comment that tells us how you think you did—you might answer one or more of the questions above.

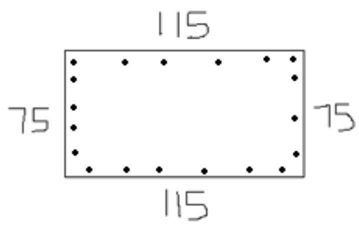
Our Solutions

Method 1: Draw a Picture

We read the problem and decided to draw a picture to think more about what to do. The field is 75 yds wide and 115 yds long and so we drew a rectangle and labeled those numbers:



We talked about where the team would be running if they ran around the field. We put dots to show where we were thinking they might be on the field.



One time around the outside of the field is 115 + 75 + 115 + 75 = 380 yards. Since we have to find out how many laps of the field would equal more than a mile, we looked up how many yards equal a mile. If 1,760 yards = 1 mile we decided to see what happened when we started adding 380s together.

(2 laps) 380 + 380 = 760 (not enough yet)
(3 laps) 380 + 380 + 380 = 1140 (not enough yet)
(4 laps) 380 + 380 + 380 + 380 = 1520 (not enough yet)
(5 laps) 380 + 380 + 380 + 380 + 380 = 1900 (it's over!)

If they ran 5 laps around the field it will be at least a mile (unless they cut the corners).

Method 2: Make a Table and Working Backwards

After I read the problem, I looked up some information and found out that a mile is 5,280 feet or 1,760 yards. I know that a lap is the perimeter of the field. I can find the perimeter if I add 115+75+115+75. I know the perimeter is 380 yards. I made a table to keep track of the numbers as I subtracted laps from a mile to find out how many a team member needed to run.

1760 = number of yards in a mile	**yards left to run**
1760 - 380	1380
1760 – 380 - 380	1000
1760 – 380 – 380 - 380	620
1760 – 380 – 380 – 380 - 380	240
1760 – 380 – 380 – 380 – 380 - 380	0 for first mile and 140 in next mile

Running 5 laps of the field is more than a mile.

Extra: Each team member runs 6 laps (= 4 + 2). Together all 22 members run 132 laps (= 6 x 22). A lap is 380 yards. The team runs 50,160 yards (=132 laps x 380 yards per lap). 50,160 yards is 28.5 miles (=50,160 yards/1,760 yards per mile).

Method 3: Work with Feet instead of Yards

First I converted the yards to feet. I knew that there are 3 feet to a yard and from the problem I knew the dimensions of the field.
75 yards = 225 feet
115 yards = 345 feet

Next I doubled both numbers because the field has four sides (2 widths and 2 lengths) and added them together to get the total perimeter of the field. That got me to 1140 feet. I know that there are 1140 feet in one lap. There are 5280 feet in a mile and I divided 5280 by 1140 and got 4.631578947, which is how many laps it takes to run exactly a mile. I then rounded this up to 5, getting 5 laps. To check this I multiplied 1140 feet in a lap by 5 laps and I got 5700 feet. Since 5700 feet is over a mile, I knew that answer is right.

Method 4: Proportional Reasoning

To get our answer we first found the perimeter of a field 75 yards wide and 115 yards long. We added 75 + 115 + 75 + 115 and got 380 yards. We then thought that

$$\frac{\text{the number of yards in a mile}}{\text{the perimeter of the field}}$$

should give us the number of laps to run since one lap is the same as the perimeter. It's not perfect, of course, because they're probably not going to run on the exact edge of the field but we think it will be close enough.

$$\frac{1760}{380} = 4.63157...$$

If each team member ran at least 5 laps then they'd run more than a mile.

Extra: The team would run 28.5 miles every day. I got this using:

$$\frac{22 \cdot (4 + 2) \cdot 380}{1760}$$

The 22 is for the 22 players. They each run a total of 6 laps (4 before practice and 2 after practice), and each lap is 380 yards.

Standards

If your state has adopted the Common Core State Standards, you might find the following alignments helpful.

Grade 3: Measurement & Data

Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Grade 4: Measurement & Data

Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

Grade 5: Measurement & Data

Convert like measurement units within a given measurement system.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Teaching Suggestions

When this problem was first offered, some students solved it using feet as their unit of measure and others stayed with yards as stated in the problem. As I read through the solutions it occurred to me that this problem could lead to some interesting conversations. Different groups of students could present their ideas and then make connections to show why each of them work out. I can imagine that in a class that might include some groups comfortable working with feet as their unit, others preferring to use yards, some adding to get to a mile and others subtracting to find out the number of laps.

What I wondered the most is how students might think about the accuracy of the situation. We're assuming that measuring the perimeter gives us the distance of the laps but would someone really run that distance or would it be a little less? Does this problem require students to be extremely accurate or is the question presented to allow for the inaccuracy of the posed situation? If you use our Noticing/Wondering activity these types of questions might surface as students are wondering!

Resist the urge to give direct instructions on a specific approach.

The questions in the Answer Check, above, might serve as good prompts to help students make progress. Encourage students to use a strategy that works for them. You can see from the various methods that we have thought to use for this problem, there are several ways to approach this problem. And keep in mind that we may not have thought of them all!

Sample Student Solutions – Focus on *Interpretation*

In the solutions below, I've provided scores the students would have received in the **Interpretation** category of our scoring rubric. My comments focus on areas in which they seem to need the most improvement.

Novice	Apprentice	Practitioner	Expert
Understands few of the concepts listed in the Practitioner column.	Understands most but not all of the criteria listed in the Practitioner column.	Understands • the goal is to find out how many laps around a field are more than a mile in distance. • yards can be converted to miles (1760 yards = 1 mile). • the distance around the field is found by adding two lengths and two widths (the perimeter).	Is at least a Practitioner in Strategy and comes up with the correct solution for the Extra.

Joshua, age 12, Novice

I think the answer is 150 laps. I mutplied 115 and 75 together, and I got 150.

I notice that Joshua decided to multiply the two numbers given in the problem. He might have had a reason or he might have just picked an operation to try.

Instead of asking him about why he multiplied I might start by asking him to describe a lap. What picture does he see in his mind if he is watching someone run a lap around a field?

Liz and Becky, age 11, Novice

We came up with 45 as our answer.

First we took 5200 yards(yardes in a mile)and divided it by 115 yards and came up with 45.

I notice that Liz and Becky are focused on the 115 yards. It makes me wonder if their idea of a lap is running back and forth from one short side and back to the other.

I might ask them to draw a picture and describe to me the path of the runner.

Kaily, age 11, Apprentice

They would need to run 41 full laps and one of the longer sides and 5 yards on one of the shorter sides.

First I found out one mile is 5280 feet. So then I had to change that to yards so I multiplyed 5280 by 3 and got 15840 yards. then I had to find out how big the soccer fild was all together i am not finshed i will come back later.

Kaily mentions that she's not finished but, as with Liz and Becky, I'm not sure how she's thinking about a lap.

I notice that Kaily multiplied the number of feet by 3 to get a number of yards. I might ask her which is larger, a foot or a yard? She might also try a smaller number like 9 feet to think about how to get from feet to yards.

Natalie, Kyle, Dani and Abby, age 11, Apprentice

ly would take 3 laps to complete more than a mile.

First we figured out that there are 1,760 yards in a mile. then we took one side of the field (115 yds) and times it by 2 after that unsucssecful try on both sides together. We tried to multiply it by 3. That worked on both sides to add of to a total of 3 laps to atleast comlete one mile.

Bonus:The team would run a total of 44 miles together. And we figured that out by how many six miles would be then times it by 22.

I notice that this group has a good start in noting how many yards are in a mile. It's great that they include their unsuccessful try but I'm not too sure why they decided it didn't work.

I wonder if they could explain more about that. Their additional explanation might uncover for me what they're considering to be a lap. My guess is that might be part of the misunderstanding.

Julia, age 10, Practitioner

5 laps around the soccer field until you run over a mile

Step 1: Draw a field

Step 2: Find out how many feet in a mile?

5280 feet

Julia has done a nice job of presenting her explanation in very clear steps.

I would ask her to consider working on the Extra.

Step 3: Add all "the yard" together.

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  75
  75
 115
+115
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 380
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Step 4: Change 5280 ft. into yards
5280 divided by 3 = 1760

Step 5: Divide 1760 by 380 = 4.6315789 or 5 laps

Reina and Chanelle, age 12, Practitioner

After solving the problem we found out that they would have to run 5 laps in order to complete over a mile.

After reading the question, we figured that we needed to find out how many yards are in a mile. After doing some research we found that there are 1,760 yards in a mile. So we thought if we multiplied 75 by 2 and 115 by 2 and added the 2 answers together we would find out how many yards there are in the soccer field. The answer was 380 so we decide to divide 1760 by 380. After dividing the two numbers we got the answer 4 but it wasn't even so we kept on going. Our final answer came out to be 4.6. So we figured in order to run over 1 mile a team member would have to complete 5 laps.

Reina and Chanelle have also done a nice job with their explanation. I might suggest that they break their one long explanatory paragraph into two parts.

I would also suggest that they might try the Extra.

Keturah, age 11, Expert

I figured out that it would take five laps to make a little over a mile.

First I added all four sides of the field. The field is 75 yards wide and 115 yards long. Since the field has four sides I added 75+ 115+75+115. That equaled 380. Next I found out that 1 mile equaled 1,760 yards. Then I multiplied 380 by 4. I got 1,520 yards. That did not make a mile yet, so added another lap (380 x 5). This equaled 1,900 yards. I figured out that it would take 5 laps to make a little over a mile.

Bonus: First I found that it would take 5 laps to make a little over 1 mile on the field. The team ran 6 laps. Then I multiplied 380 by 6. This equaled 2,280. Next I divided 1,760 into 2,280. This equaled 1.29. I figured out that the team ran 1.29 miles

Although I would score Keturah's work as Expert in Interpretation, I would encourage her to make her explanation a little more complete.

As I read it I get the feeling that she's telling me what she did but not necessarily why she did it.

Scoring Rubric

A **problem-specific rubric** can be found linked from the problem to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work.

We hope these packets are useful in helping you make the most of the Math Fundamentals Problems of the Week. Please let me know if you have ideas for making them more useful.

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