Math Fundamentals PoW Packet
Cat Walk

Problem 673 • **https://www.nctm.org/pows/**

Welcome

This packet contains a copy of the problem, the "answer check," our solutions, some teaching suggestions, and samples of the student work we received in April 1999. The text of the problem is included below. A print-friendly version is available using the "Print" link on the problem page. .

In *Cat Walk* students figure out how many feet the cat covered given the number of dog steps compared to cat steps and the length of each dog step. The **key concept** is proportional reasoning.

Standards

If your state has adopted the Common Core State Standards, this alignment might be helpful. *Grade 3: Operations & Algebraic Thinking*

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Grade 4: Operations & Algebraic Thinking

Generate and analyze patterns.

Grade 4: Number & Operations–Fractions

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

Grade 5: Operations & Algebraic Thinking

Analyze patterns and relationships.

Grade 5: Number & Operations–Fractions

5.NF.2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.

The Problem

Cat Walk

A dog takes three steps to walk the same distance for which a cat takes four steps. Suppose one step of the dog covers ½ foot. How many feet would the cat cover in taking 24 steps?
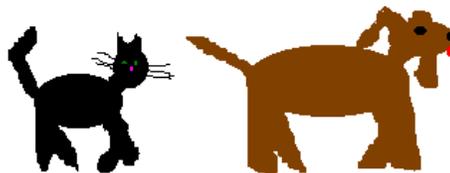
Answer Check

After students submit their solution, they can choose to "check" their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually **get** the answer) we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

> The cat would walk 9 feet in 24 steps.

If your answer **doesn't** match ours,

- did you think about how many feet the dog walked in 3 steps?
- did you make a chart with columns of dog steps, cat steps, and distance?
- did you check your arithmetic?

If any of those ideas help you, you might *revise* your answer, and then leave a comment that tells us what you did. If you're **still stuck**, leave a *comment* that tells us where you think you need help.

If your answer **does** match ours,

- have you clearly shown and explained the work you did?
- did you explain any patterns or insights you discovered while solving the problem?
- can you solve the problem using another strategy?

Revise your work if you have any ideas to add. Otherwise leave us a *comment* that tells us how you think you did—you might answer one or more of the questions above.

Our Solutions

Method 1: Make a Table

We read the problem and listed what we noticed:

- the dog and the cat both take steps
- three dog steps is the length of four cat steps
- the dog covers more distance than the cat
- one dog step covers a half foot
- the cat is going to walk 24 steps

We wondered if we could make sense of our information by making a table. We knew that ½ can also be written as .5. We also knew that if 1 dog step was .5 feet in length than 3 dog steps was three times .5 feet or 1.5 feet.

dog steps	cat steps	distance
3	4	3 x .5 = 1.5 feet
6	8	6 x .5 = 3 feet
9	12	9 x .5 = 4.5 feet
12	16	12 x .5 = 6 feet
15	20	15 x .5 = 7.5 feet
18	24	18 x .5 = 9 feet

From our table information we could see that the cat walked distance of 9 feet.

Method 2: Act It Out

After reading the problem our group decided to try to act it out. One of us played the role of the dog and another played the role of the cat. The "dog" took 3 steps and the "cat" took 4 steps. From the problem we knew that each "dog" step was 1/2 foot long. We thought about how long 3 "dog" steps would be and knew it would be

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2} \text{ foot}$$

And, that was also how far our "cat" walked in her 4 steps.

From the problem we knew the cat in the problem walked 24 steps and so our "dog" and our "cat" continued walking. Each time they walked 3 "dog" steps or 4 "cat" steps we added on another 3/2 foot. They walked until we counted to 24 "cat" steps. Then we just had to add the list of 3/2 that we'd made. We had:

$$\frac{3}{2} + \frac{3}{2} + \frac{3}{2} + \frac{3}{2} + \frac{3}{2} + \frac{3}{2}$$

$$= \frac{18}{2}$$

$$= 9 \text{ feet}$$

If the cat walks 24 steps, she covers 9 feet.

Method 3: Draw a Picture

As my teacher read the story of the problem I could see the dog and the cat walking along and I decided to draw a picture to see what was happening. This is what I drew:
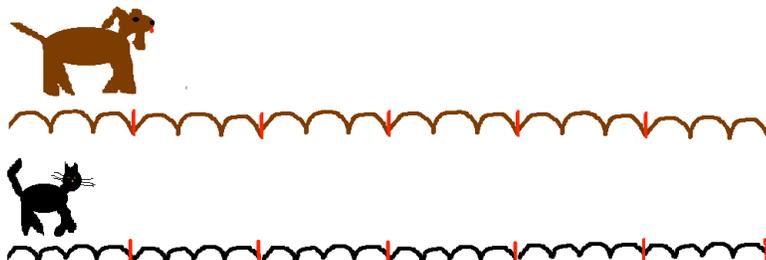

I made three loops for the dog and marked it and then I made four loops to show the steps the cat made. Usually cats are smaller than dogs so it made sense to me that it would take a cat more steps to cover the distance a dog walked. I wasn't too sure about a cat and a dog walking side-by-side like they're friends … but … maybe!

Now I just had to think about how far that was. I labeled my picture with what I knew about the dog. One dog step was a ½ foot.
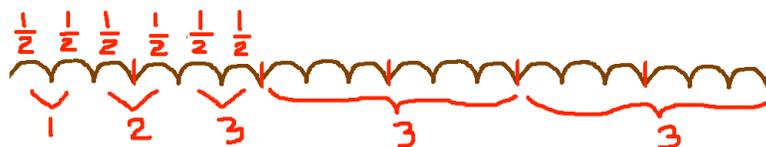
I started labeling each dog step with a ½ but then I saw that ½ + ½ = 1. And then I could see that I could know the 6 more steps would be 3 feet. In the end I counted 9 feet. Since the cat's 24 steps would be the same distance as the dog's 18 steps, I knew that the cat covered 9 feet.
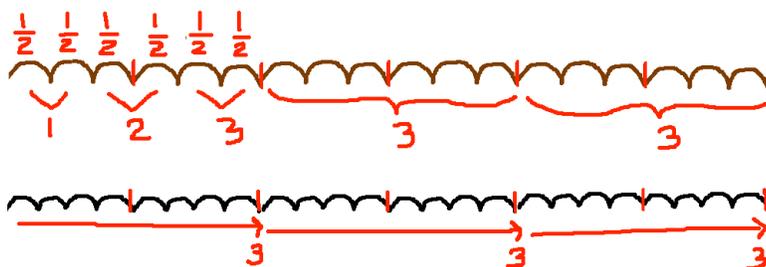
Method 4: Algebra

We know from the text of the problem that the cat would take 24 steps. We also know that each time the cat takes a total of 4 cat steps the dog takes a total of 3 dog steps. So the ratio of cat steps to dog steps is 4:3. Let the number of dog steps that is equivalent to 24 cat steps be x. Note that

4:3 = 24:x

We can write that as,

$$\frac{4}{3} = \frac{24}{x}$$

and we can solve for x,

$$3x\left(\frac{4}{3}\right) = 3x\left(\frac{24}{x}\right)$$

$$\frac{4x}{1} = \frac{72}{1}$$

$$4x = 72$$

$$\frac{4x}{4} = \frac{72}{4}$$

$$x = 18$$

So the number of dog steps that is equivalent to 24 cat steps is 18. Since each dog step is half a foot, the number of feet that would be covered by the cat in 24 cat steps is 18 · ½, which is 9. So the cat would cover 9 feet.

Teaching Suggestions

When we first offered this problem there were a variety of approaches, including finding the distance for four cat steps, setting up algebraic equations, drawing a picture, making charts or tables or lists, using manipulatives to act out the dog and cat activity, and finding equivalent ratios. Depending on the sophistication of the problem solver the methods ranged from very concrete to more symbolic or abstract.

The questions in the Answer Check, above, might serve as useful prompts to help students make progress. Encourage students to use a strategy that works for them. You can see since there is more than one method that we have thought to use for this problem, there may be several ways to approach this problem.

If you would like a calendar of the Current Problems, consider bookmarking this page:

Sample Student Solutions

focus on **Interpretation**

In the solutions below, I've provided the scores the students would have received in the **Interpretation** category of our scoring rubric. My comments focus on what I feel is the area in which they need the most improvement.

Novice	Apprentice	Practitioner	Expert
Understands few of the criteria listed in the Practitioner column.	Understands most but not all of the criteria listed in the Practitioner column.	Understands that • the goal is to figure out how many feet the cat covered in 24 steps. • dog steps**:**cat steps is 3**:**4 (although they might note this more informally and not use ratio notation). • one dog step covers a half foot.	There is no Extra and no way to be an Expert in this category for this problem.

Brandayn
age 12

Interpretation
Novice

I think the cat walked 48 feet

Well I used what we learnd in math today.

I notice that Brandayn found a number of feet that the cat walked which is what the question asked but since I wasn't in the math class I'm not too sure what was learned. That's probably how I'd start my conversation. Since I wasn't there, please try to tell me what happened!

Brian

age 11

Interpretation

Novice

The cat will go 20 feet if he takes

1. I found out that he moved 10 inches for every foot.

2. I multiplied 10 times 24 equals 240.

3. I divide 240 by 12 which equaled 20.

I noticed Brian has organized his three steps. I wonder how he thought of each of the statements he's made. What information from the problem did he use to find out the cat moved 10 inches for every foot? I wonder what that really means since inches and feet are both linear measurements. I wonder if drawing a picture or acting it out might help Brian.

Wesley

age 9

Interpretation

Apprentice

I found out how many feet a dog takes in 3 steps and multiplyed that

times 6 and got my answer.

4.5

x 6

_____

27.0

I read a little between the lines to score Wesley as an Apprentice in Interpretation although I could also decide to score this as a Novice.

I noticed his first statement that he found out how many feet the dog takes in 3 steps. I wonder what number he multiplied by 3 to get 4.5. Was that a number given in the problem?

Jared

age 11

Interpretation

Apprentice

The answer to this problem is 108.

First I multiplied 1/2 ft. by 3 and got 1 1/2 ft. Then I converted 1

1/2 ft. to 18in. Then I multiplied 18 by 6 and got 108. That is how I

got the answer to this problem.

Jared followed steps similar to Wesley's, although he added in the complication of changing feet to inches. And, he got 1 1/2 instead of 4 1/2. In both cases they multiplied by 6 as their final step. I wonder what that final number represents. Does it refer to the cat? How?

Hovey

age 9

Interpretation

Apprentice

My calculations show that if the dog walks 18 steps which is 9 ft.

4 steps of cat = 3 steps of dog

1 step of cat = 3/4 steps of dog

24 steps of cat = 18 steps of dog

1 step of dog= 1/2 ft

18 steps of dog = 9 ft

I notice that Hovey has thought about some useful comparisons. It would be helpful if he told us just a little more about what he was thinking.

His answer refers to the dog rather than the cat. I would ask him to take one more look at the actual question.

Danielle
age 10

Interpretation
Practitioner

The answer is 9.

	Catwalk.
cats	
4	1 ½
8	3
12	4 ½
16	6
20	7 ½
24	9

The answer is   9

HOW WE DID IT.

The first column we did it in fours.

Danielle's chart is helpful if you already know what's going on with this problem. I can see that she has interpreted it correctly. Her chart also makes her explanation clear.

It might help her improve her Completeness score if she tried to think of a classmate reading her solution to understand. Can she add some details to explain why she made her chart and how she concluded what she did?

Team Forloines
average age 8

Interpretation
Practitioner

The cat coverd 9ft. taking 24 steps.

          cat

1
2
3
4_____ 1 1/2
5
6
7
8_____1 1/2 or 3 total
9
10
11
12_____1 1/2 or 4 1/2 total
13
14
15
16_____ 1 1/2 or 6 total
17
18
19
20_____1 1/2 or 7 1/2 total
21
22
23
24_____1 1/2   or 9 total feet   In 24 steps the cat goes 9 feet. Since one step of the dog is 1/2 foot, 3 dog steps are 1 1/2 feet. The cat takes 4 steps to get 1 1/2 feet. So in 24 steps it goes 9 feet.

This team has made a nice chart and also explained their thinking. It still might benefit from a little more explanation but the Interpretation score is at the Practitioner level.

Katelyn

age 10

Interpretation
Practitioner

The cat walked 9 feet in 24 steps.

To solve this problem, I broke it into pieces.
First, I figured out that each cat step covered 4 1/2 inches, as to each dog step, which covered 6 inches each, by drawing a picture:

Dog
Start: 6" / 6" / 6" /Finish = 1 1/2 feet or 18 inches

Cat
Start: 4 1/2 \ 4 1/2 \ 4 1/2 \ 4 1/2 \Finish = 18 inches or 1 1/2 feet

As you can see, the cat's steps are smaller than the dog's, so to get to equal 18", I found that 4 1/2" worked because 4 1/2 + 4 1/2 + 4 1/2 + 4 1/2 = 18"

Now that I know how many inches the cat travels in one step, all I have to do is multiply 4.5 X 24, which gives me 108.0, and to convert it to feet I divided 12, (inches in a foot) into 108. 12 divided into 108 is 9 feet.

The cat walked 9 feet in 24 step

Katelyn has done a nice job explaining each step and using formatting to help the reader follow each step she took. Well done!

Scoring Rubric

A **problem-specific rubric** can be found linked from the problem to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work.

We hope these packets are useful in helping you make the most of Math Fundamentals Problems of the Week. Please let me know if you have ideas for making them more useful.

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