NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Problem of the Week Teacher Packet

Eating Contest

The students in Ms. Nomial's algebra class like to play Factor Bingo. Each student gets a bingo card, and then Ms. Nomial starts writing polynomials on the board. Students fully factor each polynomial, and then cover any of those factors that they have on their card. The first student to cover 5 factors in a row, column, or diagonal wins the game.

After watching the famous hot dog eating contest at Coney Island this summer, four kids decided to have their own contest to see who could eat the most green beans in five minutes.

Caleb ate an impressive number of beans, but Josh ate eight more than Caleb did. Elsie ate fifteen fewer than twice as many as Caleb did. Sol's total was thirty more than half of Caleb's.

If the children ate a total of 176 beans, what was the finishing order of the contest and how many green beans did the winner eat?

Extra: If the green bean relationships between the kids stay the same, what's the smallest total number of beans eaten that would result in a tie for first place in the contest?

Answer Check

After students submit their solution, they can choose to "check" their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually get the answer) we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

Elsie ate the most green beans. (Be sure that your answer also includes how many beans Elsie ate and the finishing order of all four kids.)

If your answer does not match our answer,

- did you try picking a number of beans for Caleb and calculating the resulting number of beans for each of the other children?
- did you try expressing the number of beans Caleb ate as a variable and then writing expressions using that variable for each of the other kids?
- did you keep in mind that the total number of beans eaten by all four kids was 176?
- did you check your arithmetic?

If any of those ideas help you, you might revise your answer, and then leave a comment that tells us what you did. If you're still stuck, leave a comment that tells us where you think you need help.

If your answer does match ours,

- did you use algebraic techniques to find your answer?
- did you show and explain the thinking and work you did?
- is your explanation clear and complete? Would another student understand your solution?
- did you make any mistakes along the way? If so, how did you find and fix them?
- are there any hints that you would give another student?
- have you tried the Extra question?

Revise your work if you have any ideas to add. Otherwise leave us a comment that tells us how you think you did—you might answer one or more of the questions above.

Our Solutions

I noticed that all four kids are expressed in terms of Caleb, so I decided to use a variable to represent how many beans Caleb ate. Then I wrote expressions for the other three kids with that same variable based on what the problem said about them, so I let:

c = the number of green beans Caleb ate
c + 8 = the number of green beans Josh ate
2c – 15 = the number of green beans Elsie ate
0.5c + 30 = the number of green beans Sol ate

I know that together the four kids ate a total of 176 beans, so I wrote an equation adding them up:

$$c + c + 8 + 2c - 15 + 0.5c + 30 = 176$$

I combined like terms on the left side:

$$4.5c + 23 = 176$$

I subtracted 23 from each side and then I divided both sides by 4.5 to get the answer for c:

$$4.5c = 153$$
$$c = 34$$

Now I know that Caleb ate 34 green beans. I substituted 34 into the expressions for the other kids to figure out how many each of them ate:

$$Caleb = c = 34$$
$$Josh = c + 8 = 34 + 8 = 42$$
$$Elsie = 2c - 15 = 2(34) - 15 = 68 - 16 = 53$$
$$Sol = 0.5c + 30 = 0.5(34) + 30 = 17 + 30 = 47$$

I checked my answer to see if the total for the four kids was 176, and it was:

$$34 + 42 + 53 + 47 = 176$$

Elsie ate the most beans with 53. Sol was next with 47, Josh was third with 42, and Caleb was fourth with 34.

Method 2: Make a Mathematical Model (Multiple Variables)

I picked a different variable for each of the four kids, I let:

$$c = the\ \ number\ \ of\ \ green\ \ beans\ \ Caleb\ \ ate$$
$$j = the\ \ number\ \ of\ \ green\ \ beans\ \ Josh\ \ ate$$
$$e = the\ \ number\ \ of\ \ green\ \ beans\ \ Elsie\ \ ate$$
$$s = the\ \ number\ \ of\ \ green\ \ beans\ \ Sol\ \ ate$$

I know that the four kids ate a total of 176 beans, so I wrote an equation:

$$c + j + e + s = 176$$

I used the information given in the problem to write equations for each kid. For example, I know Josh ate 8 more than Caleb, so I wrote j = c + 8. I came up with:

Caleb = c
Josh ate 8 more than Caleb so: $j = c + 8$
Elsie ate 30 fewer than twice Caleb so: $e = 2c - 30$
Sol ate 15 more than half of Caleb so: $s = 0.5c + 15$

I substituted the individual equations into my main equation:

$$c + j + e + s = 176$$
$$c + (c + 8) + (2c - 30) + (0.5c + 15) = 176$$

I combined like terms on the left side:

$$4.5c + 23 = 176$$

I subtracted 23 from each side and then I divided both sides by 4.5 to get the answer for c:

$$4.5c = 153$$
$$c = 34$$

Now I know that Caleb ate 34 green beans. I substituted 34 into the expressions for the other kids to figure out how many each of them ate:

$$Caleb = c = 34$$
$$Josh = c + 8 = 34 + 8 = 42$$
$$Elsie = 2c - 15 = 2(34) - 15 = 68 - 16 = 53$$
$$Sol = 0.5c + 30 = 0.5(34) + 30 = 17 + 30 = 47$$

I checked my answer to see if the total for the four kids was 176, and it was:

$$34 + 42 + 53 + 47 = 176$$

Elsie ate the most beans with 53. Sol was next with 47, Josh was third with 42, and Caleb was fourth with 34.

Method 3: Guess and Check

I started by guessing that Caleb ate 30 beans. Then I wrote calculations for the other three kids, but I did not calculate them out right away:

Caleb ate 30.
Josh ate 30 + 8.
Elsie ate 2(30) - 15
Sol ate (30/2) + 30

I know that the four kids need to total 176, so I wrote an equation and checked to see if it was right:

$$Caleb + Josh + Elsie + Sol = 176$$
$$30 + (30 + 8) + 2(30) - 15 + (30/2) + 30 = ?176$$
$$30 + 38 + 45 + 45 = ?176$$
$$158 \neq 176$$

So, that means my guess of 30 for Caleb was too low.

I started by guessing that Caleb ate 30 beans. Then I wrote calculations for the other three kids, but I did not calculate them out right away:

Caleb ate 40.
Josh ate 40 + 8.
Elsie ate 2(40) - 15
Sol ate (40/2) + 40

I know that the four kids need to total 176, so I wrote an equation and checked to see if it was right:

$$Caleb + Josh + Elsie + Sol = 176$$
$$40 + (40 + 8) + 2(40) - 15 + (40 / 2) + 40 = ? \ 176$$
$$40 + 48 + 65 + 40 = ? \ 176$$
$$193 \neq 176$$

So, that means my guess of 40 for Caleb was too high.

I can see that I put 40 in all the places I put 30 when I was testing those numbers. I think I can go back to my original equation and replace all the 30's for Caleb with a variable. I'll use c. That will help me get an algebraic equation to solve the problem:

$$Caleb + Josh + Elsie + Sol = 176$$
$$30 + (30 + 8) + 2(30) - 15 + (\frac{30}{2}) + 30 = 176$$
$$c + (c + 8) + 2(c) - 15 + (\frac{c}{2}) + c = 176$$

Now that I have an equation, I combined like terms on the left side, and isolated the variable:

$$4c + \frac{c}{2} + 23 = 176$$
$$8c + c + 46 = 352$$
$$9c + 46 = 352$$
$$9c = 306$$
$$c = 34$$

Now I know that Caleb ate 34 green beans. I substituted 34 into the expressions for the other kids to figure out how many each of them ate:

$$Caleb = c = 34$$
$$Josh = c + 8 = 34 + 8 = 42$$
$$Elsie = 2c - 15 = 2(34) - 15 = 68 - 16 = 53$$
$$Sol = 0.5c + 30 = 0.5(34) + 30 = 17 + 30 = 47$$

I checked my answer to see if the total for the four kids was 176, and it was:

$$34 + 42 + 53 + 47 = 176$$

Elsie ate the most beans with 53. Sol was next with 47, Josh was third with 42, and Caleb was fourth with 34.

Standards

If your state has adopted the Common Core State Standards, this alignment may be helpful:

Algebra: Creating Equations

1. Create equations and inequalities in one variable and use them to solve problems.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Algebra: Reasoning with Equations and Inequalities

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method..

Algebra: Reasoning with Equations and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning

Solve equations and inequalities in one variable.

Algebra: Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.

Teaching Suggestions

In *Eating Contest*, students have an opportunity for students to use variables and an algebraic representation or to start with guess and check and use that as a starting point for developing an algebraic representation. This problem was originally constructed to focus on the Guess and Check Strategy and how it can be used to gain a sense of the problem, as well as providing an access point for the use of algebra in the solution.

This problem is accessible both to students with an understanding of how to write variable expressions and equations and to students who don't know those algebraic methods but can use Guess and Check to solve it. It's a nice opportunity to expose students to the power of algebra as they make the transition from Guessing and Checking to using a variable and being able to go directly to the correct solution without the need for multiple attempts.

One activity that might help those students who are not yet familiar or comfortable with using variables would be to give students two numbers and have them write their own comparisons between the numbers. For example, if given 6 and 28, a student might say that 28 is two fewer than five times six. Another student might say that six is one fewer than one-fourth of 28, and so on. They might start by making these as verbal or written statements without worrying about writing out any math. Then they could move to writing the same statements mathematically, so that $28 = 5(6) - 2$ and $6 = 28/4 - 1$.

Next, ask them to find another pair of numbers that fits the same relationship they stated for 6 and 28. For instance, the student who said that 28 is two fewer than five times six might come up with 18 and 4, where 18 is two fewer than five times four.

From there, have them choose two variables to represent two numbers and write an equation relating those two numbers that maintains the same relationship they've been using. Continuing the example, the student would now write that $a = 5(b) - 2$. Finally, have them use their equation to find three more pairs of numbers that fit their relationship.

For those students who are somewhat comfortable with variables already, this activity also serves as a reminder that a two-variable equation is simply a mathematical representation of a relationship between two quantities.

The Online Resources Page for this problem contains links to related problems in the Problem Library and to other web-based resources.

Sample Student Solutions – Focus on Strategy

In the solutions below, I've provided the scores the students would have received in the **Strategy** category of our scoring rubric. The table below is an excerpt from the rubric for this problem showing the guidelines for scoring in Strategy. My comments focus on what I feel is the area in which they need the most improvement.

Novice	Apprentice	Practitioner	Expert
Has no ideas that will lead them toward a successful solution. Has not written enough to tell what strategy they might have used.	Picks an incorrect strategy, or relies on luck to get the right answer. For example, happens to guess 34 for Caleb but does not provide any reasoning behind the guess or how he/she knew it worked.	Picks a sound strategy—success achieved through algebraic skill, not luck. Uses an appropriate strategy, like uses Guess and Check in an organized fashion or uses variable(s) to represent the various kids and writes and solves an equation.	Uses two separate strategies or an unusual or sophisticated strategy. For example, might first solve using Guess and Check and then solve using an algebraic technique.

John, age 14, Novice

the total of the bean is 176 so it needs to get more likely if too much could get more beans

176-x=20

x=20-176=-156

other solution

x=20+156= 176.

John's set up an algebraic equation, but it's not obvious what his x represented or what the 20 represents. I'd begin by asking John to tell me the story of this problem. Then I'd encourage him to organize what he knows about the story – what's happening and to whom is it happening.

Reynold, age 15, Novice

The winner is Elsie.

Else has the most beans because 176 - 15.

Reynold has shared who he proposes the winner to be and not much else. The equation he offers does not reveal a lot about his strategy, so I'd begin by asking him to tell me what he learned from subtracting 15 from 176 and how he knew that Elsie was the winner.

Mitchell, age 12, Apprentice

So, the finishing order of the contest is Elsie with 53 beans, Sol with 47 beans, Josh with 42 beans, and Caleb with 34 beans perspectively.

I started my problem by organizing it so I would know how each result related to each other:

Caleb: ate a good number

Josh: ate 8 more than Caleb

Elsie: x2 -15 of Caleb

Sol: 30 more than half of Caleb

After that I started a guess and check process:

Caleb:	50	44	30	36	34
Josh	58	52	38	44	42
Elsie:	85	73	45	57	53
Sol:	55	52	45	48	47

The first four were just trials and the fifth one was my solution.

Mitchell has nicely organized and communicated his problem solving strategy. My goal for Mitchell would be to transition from this organized strategy to the use of variable. I would begin by asking him to look across his descriptions of how many each kid ate and ask what he noticed was the same in all of them. Assuming he notices Caleb, I'd ask him to try using a variable to represent the number Caleb ate.

Fauzy, age 15, Apprentice

Finishing order are (from first place until fourth) Elsie, Sol, Josh and Caleb and the winner is Elsie who ate 53 beans

Total children = 4
Total green beans eaten = 176

Caleb = 34
Josh = 34 + 8 (eight more that Caleb's) = 42
Elsie = 2 x 34 - 15 (fifteen fewer than twice as Caleb's did) = 53
Sol = Caleb / 2 + 30 (thirty more than half of Caleb's) = 47
Total = 176 beans

- The finishing order of the contest :
  First Place = Elsie (53 beans)
  Second Place = Sol (47 beans)
  Third Place = Josh (42 beans)
  Fourth Place = Caleb (34 beans)
- The winner is Elsie who eat **53** beans in 5 minutes

Fauzy's got the correct answer, but it's very hard to know how that answer was arrived at. It's clear from the way he has calculated the other kid's totals, using the value of 34 for Caleb that he has interpreted the relationships well and I'd try to use that as a starting point. I'd ask Fauzy to show me how he'd represent each kid's total if he didn't know Caleb ate 34 beans.

Felicia, age 15, Practitioner

1st winner: Sol, 2nd:Josh, 3th:Caleb, Elsie Sol ate 37 beans

let's just say caleb is X
it says that Josh ate eight more than Caleb did (8 + X)
Elsie ate 2 times caleb did, and ate fifteen fewer (2X - 15)
Sol's ate half of caleb did, and ate thirty more than half of Caleb's. (1/2X + 30)

and just add all of those things
X + 8X + 2X -15 +1/2X +30= 176
11 1/2X +15= 176
11 1/2X= 176 - 15
11 1/2X= 161
X= 161/11 1/2
X= 14

So caleb ate 14 beans (third)
Josh ate 8 + 14 = 22 (second)
Elsie 28 - 15 = 13 (fourth)
and Sol = 7 + 30 = 37 (first)

Felicia's strategy is apparent and her explanation shows both completeness and clarity at the Practitioner level as well. Her only issue seems to be that she's set up her equation incorrectly. I'd begin by asking her how she'd check her solution, and then ask her to circle the number of beans each kid ate in her original equation.

Larry, age 13, Practitioner

The number of green beans eaten by Caleb was 34, Josh was42, Elsie was 53,and Sol was 47.

We define the number of green beans eaten by each student as:

-Caleb "x"

-Josh "x+8"

-Elsie "2x-15"

-Sol "x/2 + 30"

The problem tells us x + (x+8) + (2x-15) + (x/2 + 30) = 176

So 41/2x+23 = 176

Multipllyimg by 2 we have 9x +46 = 352

9x = 306

So x = 34

Thus, Caleb eats 34 green beans, Josh eats 42 green beans, Elsie eats 53 green beans and Sol eats 47 green beans.

To check we compute the total 34+ 42+ 53+ 47 = 176.

Larry has shared his solution in a way that is easy to follow and nicely algebraic. Unlike many of the other examples here, he has also checked his work which can be an important step in confirming for yourself that your solution is accurate, I'd commend Larry on taking that step. I'd also encourage him to try the extra.

Scoring Rubric

A **problem-specific rubric** can be found linked from the problem to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work.

We hope these packets are useful in helping you make the most of Algebra Problems of the Week. Please let me know if you have ideas for making them more useful.

https://www.nctm.org/contact-us/