NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Problem of the Week Teacher Packet

Frog Farming

Farmer Mead would like to raise frogs. She wants to build a rectangular pen for them and has found 36 meters of fencing in her barn that she'd like to use.

1. Design at least four different rectangular pens that she could build. Each pen must use all 36 meters of fence. Give the length and width for each of the pens.
2. If each frog needs one square meter of area (1 m^2), how many frogs will each of your four pens hold?

Be sure to explain your strategy and your reasons for your steps.

Extra: Farmer Mead wants to build the rectangular pen that will hold the maximum number of frogs. Find the dimensions of that pen and tell how you know you're correct.

Answer Check

After students submit their solution, they can choose to "check" their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually get the answer) we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

> *There are nine different rectangles possible using whole number dimensions. For example, a 6 meter by 12 meter pen holds 72 frogs.*

If your answer does not match our answer,

- did you remember that a square **is** a special kind of rectangle?
- does your solution include at least four different rectangles?
- did you list the dimensions of each rectangle using correct units as well as the area (how many frogs it will hold)?
- did you check that all of your pens have a perimeter of exactly 36 m (the length of the fence).

If any of those ideas help you, you might revise your answer, and then leave a comment that tells us what you did. If you're still stuck, leave a comment that tells us where you think you need help.

If your answer does match ours,

- have you clearly shown and explained the work you did?
- did you make any mistakes along the way? If so, how did you find and fix them?
- are there any hints that you would give another student?
- does this problem remind you of experiences you've had?
- did you try the Extra?

Revise your work if you have any ideas to add. Otherwise leave us a comment that tells us how you think you did—you might answer one or more of the questions above.

Our Solutions

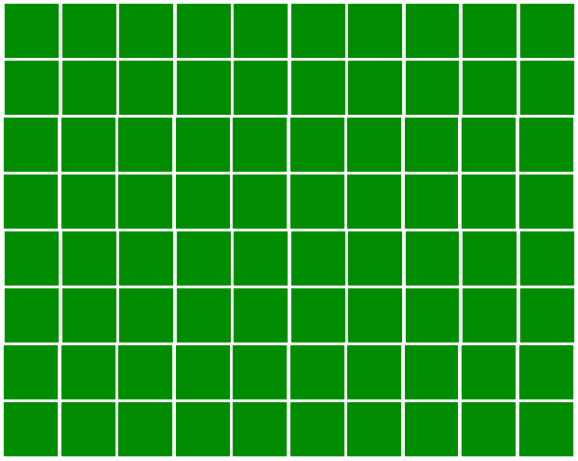
Method 1: Use Manipulatives

After reading the problem we made a model using tiles. Each tile was 1 inch by 1 inch and we decided to think of that as 1 meter by 1 meter. We counted out 10 tiles to see what that looked like.

The perimeter is the distance around the rectangle and so the top is 10 units long plus the left side is 1 more unit and then another 10 units along the bottom and 1 more unit on the right side. We had a total of 22 units and that's wasting 14 units of the total 36 the Farmer Mead wants to use.

We tried two rows of 10 tiles to see what would happen.

We noticed that the 10 units on the top and the 10 units on the bottom stayed the same but both the right side and the left side got longer. The perimeter was 24 units and now we were only wasting 12 units. We figured that if each row of 10 added 2 units more of perimeter, if we put down 6 more rows of tiles, we would have exactly enough. Our rectangle that is 8 rows of 10 has a perimeter of 8 + 8 + 10 + 10 is 36 meters. It would hold 80 frogs because 10 • 8 = 80.



Now we just have to find 3 more! We moved the tiles around and we figured out we could make a rectangle that had 7 rows with 11 in each row and that would give us a perimeter of 11 + 7 + 11 + 7 = 36. It would hold 77 frogs, we kept removing one row and making the rows one longer and found two more rectangles.

6 rows of 12 in each row. 6 + 6 + 12 + 12 = 36 feet of fence. 6 • 12 = 72 frogs
5 rows of 13 in each row. 5 + 5 + 13 + 13 = 36 feet. 5 • 13 = 65 frogs

Extra: We knew we had a good start on finding all of the solutions and so we made a table using what we had so far:

length	width	area (number of frogs)
10	8	80
11	7	77
12	6	72
13	5	65

We noticed a pattern in our table. The lengths are increasing by 1 as the widths are decreasing by 1. We thought we'd try going one higher for the length and one higher than the width.

length	width	area (number of frogs)
9	9	81
10	8	80
11	7	77
12	6	72
13	5	65
14	4	56

Both of those worked. We realized that if we increased the width more, we'd have an 8 by 10 and that's the same as a 10 by 8. We looked at increasing the length and saw we could probably keep going.

length	width	area (number of frogs)
9	9	81
10	8	80
11	7	77
12	6	72
13	5	65
14	4	56
15	3	45
16	2	32
17	1	17

Our chart now shows all of the possible rectangles with a perimeter of 36 meters. The one with the largest area and so will hold the maximum number of frogs is the 9 x 9.

Method 2: Logical Reasoning

I started by noticing and wondering thinking that after I made my list I would have an idea of how to get started.

I noticed:

- there's 36 meters of fencing material
- she found the fencing in the barn
- Farmer Mead wants to build a pen for frogs
- the pen is to be rectangular
- there must be at least four different possibilities
- each square meter of area is enough for one frog

I wondered:

- will the frogs jump out?
- is a square a rectangle?
- are there are more than four possibilities?

I knew that the fencing material would go around the outside of the rectangular pen and so that would be the perimeter of the rectangle. The perimeter of a rectangle is two lengths added to two widths and so the numbers would need to be even. If I found sets of even numbers that added up to 36m I could answer the first question.

I found these even numbers added to 36: 12m+24m, 14m+22m, 16m+20m, and 18m+18m. Since the two numbers can be divisible by two, I found the side length by dividing the two numbers. Then to make sure that they added up to 36, I added each of the lengths.

6m + 6m + 12m + 12m = 36m
7m + 7m + 11m + 11m = 36m
8m + 8m + 10m + 10m = 36m
9m + 9m + 9m + 9m = 36m

Question 2 asks how many frogs each pen will hold. I found that number by find the number of square meters for each pen:

6m x 12m = 72 square meters
7m x 11m = 77 square meters
8m x 10m = 80 square meters
9m x 9m = 81 square meters

So, that means that the number of frogs each pen would hold would be 72, 77, 80, and 81.

Method 3: Make a Table

A rectangle with a perimeter of 36 m must have a length and width that add up to 18, since that is half the perimeter, the sum of one length and one width. I listed all the possible pairs of whole numbers that add to 18 in a table in order from greatest to smallest length.

length	width
17	1
16	2
15	3
14	4
13	5
12	6
11	7
10	8
9	9

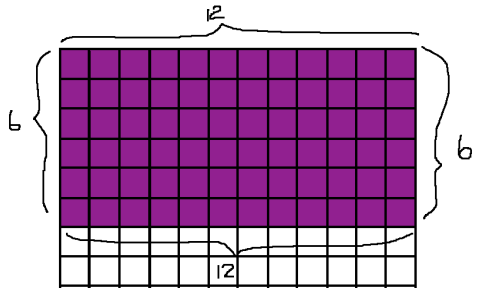
Each time I decrease the length by one, I increase the width by one to make up for it. The area of a rectangle = length • width and tells how many frogs the pen will hold. Here's what I found:

length	width	length * width
17	1	17
16	2	32
15	3	45
14	4	56
13	5	65
12	6	72
11	7	77
10	8	80
9	9	81

The pen that holds the most frogs is the 9 by 9 square. I know I have found the largest pen because I tried all the pairs of numbers that add to 18.

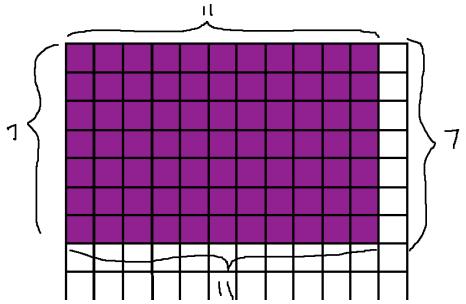
Method 4: Drawing a Picture

After I read the problem I thought about what a rectangle might look like that had a perimeter of 36 because the fencing material that Farmer Mead had would be the same as thinking about the perimeter. My drawing looked like this:



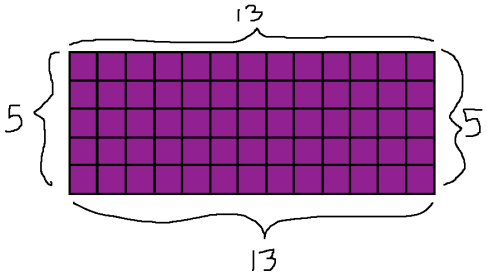
One pen could be a 12 meter by 6 meter and the number of frogs it would hold is 72. I could either count the squares or multiply 12 times 6.

I wondered what might happen if I made it one shorter in length and I found that 11 meters by 7 meters also made a rectangle with a perimeter of 36 meters.

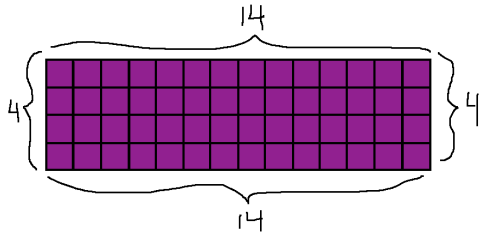


That pen would hold 77 frogs. I could either count the squares or multiply 11 times 7.

Next I tried one longer in length and I found that 13 meters by 5 meters. It holds 65 frogs.



I just needed one more. I made it one longer and I found that 14 meters by 4 meters had a perimeter of 36. It holds 56 frogs.



Standards

If your state has adopted the Common Core State Standards, you might find the following alignments helpful.

Grade 3: Measurement & Data

> Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Grade 4: Measurement & Data

> Apply the area and perimeter formulas for rectangles in real world and mathematical problems.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.

Teaching Suggestions

Frog Farming requires understanding the concepts of area and perimeter, specifically as applied to rectangles. Some children will be familiar with formulas but may not be able to apply them in this situation. Children without formal skills may solve the problem by using manipulatives and/or graph paper and a systematic guess and check strategy. Critical to any successful solution is the understanding that there are at least four different rectangles with perimeter of 36 meters. The areas of the rectangles (and therefore capacity – the number of frogs that can be sustained) vary from 17 m^2 to 81 m^2. We've not listed any fractional dimensions in our solutions (above) but they are acceptable as long as the resulting shape follows all the rules, for example 6.5 by 11.5 m. Students need to recognize that a square is a rectangle in order to successfully answer the Extra.

The first time we offered this problem we noticed some insights, including:

- The dimensions must be pairs of numbers that add to 18, half the perimeter.
- The closer the rectangles get to a square, the smaller the difference between their areas.
- The rectangles that are closer to being square hold more frogs because the area is more compacted. In the long narrow rectangles, more of the area touches the fence.

As you can see from the strategies suggested in this packet, students could approach this problem from different starting points.

The questions in the Answer Check, above, might serve as useful prompts to help students make progress. Encourage students to use a strategy that works for them. You can see since there is more than one method that we have thought to use for this problem, there may be several ways to approach this problem.

Sample Student Solutions – Focus on *Completeness*

In the solutions below, I've provided scores the students would have received in the **Completeness** category of our scoring rubric. My comments focus on areas in which they seem to need the most improvement.

Novice	Apprentice	Practitioner	Expert
Has written very little that explains how the answer was achieved.	Might show that their solution works without saying anything about how they figured it out. Might summarize their strategy without showing any math work to justify their answer.	Tells all of the important steps taken to solve the problem, which should include: • any relationships used. • the rationale behind each decision they made. • explaining why their answer is correct.	Adds in useful extensions and further explanation of some of the ideas involved. The additions are helpful, not just "I'll say more to get more credit."

Kelynn, age 11, Novice

I did the work in my head

1.The four rectangles I chose 13x5, 17x1, 12x6, and 7x11. 2.a 13x5 can fit 65 frogs, a 17x1 can fit 17 frogs, a 12x6 can fit 72 frogs, and a 7x11 can fit 77 frogs Extra: The pen that could fit the most frogs is the 9x9, which can hold 81 frogs.

I'm so curious to know what work Kelynn did in her head. I would hope to give her feedback so that she knows doing some mental math is a great idea with this problem. If she could describe what she thought about, that thinking might really help a classmate. She didn't seem to have any trouble typing her answers and so maybe if I ask nicely she'll tell me more!

Andrew, age 11, Novice

Pen #1 would be nine centimeters on each side, #2 would be 3 cm on 2 sides and 15 cm on the others.

I just divided 36 by four and got my answer

I noticed that Andrew divided the perimeter by 4. I wonder why that got him an answer. He's just written very little to explain how he found the dimensions of that first pen and nothing about how he thought about the second possibility.

I would start by asking him to tell me how he figured out the 3 cm by 15 cm pen. With time I might point out the issue with the units but not at first!

William, age 10, Apprentice

My 4 pens are 11x7, which holds 77 frogs, 10x8,which holds 80 frogs, 16x2, which holds 32 frogs, and 1x17, which holds 17 frogs. Extra - If Farmer Mead wants to build a pen to hold the maximum number of frogs, the pen would be 9x9, which would hold 81 frogs.

I designed my 4 pens - the perimeter must be 36 meters.
Perimeter = 2L +2W

Pen # 1 = 7x11 = 2x7 + 2x11 = 36 meters
Pen # 2 = 10x8 = 2x10 + 2x8 = 36 meters
Pen # 3 = 16x2= 2x16 + 2x2= 36 meters
Pen #4 = 1x17 = 2x1 + 2x17 = 36 meters

#2
If each frog needs 1 m2 of space then I need to find the area of each pen to see how many frogs will fit.
Area = L x W

Pen #1 = 7 x 11 = 77 frogs will fit in this pen
Pen #2 = 10 x 8 = 88 frogs will fit in this pen
Pen #3 = 16 x 2 = 32 frogs will fit in this pen
Pen #4 = 1 x17 = 17 frogs will fit in this pen

Extra - If Farmer Mead wants to build a pen to hold the maximum number of frogs, his pen would need to be 9x9.

William has done a great job formatting his solution so that it's very clear to read. He's also interpreted the problem correctly and done a nice job with Extra.

You may be wondering why his Completeness score is Apprentice rather than Practitioner! Although he found 4 pens that use 36 m of fencing, it would be helpful for the reader to know how, for example, he knew that a 7 m by 11 m would work. He doesn't mention if he used guess and check or some other way to figure out the dimensions of each pen. I would ask him what his strategy was.

The perimeter would be 2L+ 2W = 2x9+2x9 = 36 meters
The area would be LxW = 9x9 = 81 frogs would fit into this pen. I knew this pen would hold the most frogs because I tried out all the combinations

Pen Size	# of Frogs
1x17	36 meters holds 17 frogs
2x16	36 meters holds 32 frogs
3x15	36 meters holds 45 frogs
4x14	36 meters holds 56 frogs
5x13	36 meters holds 65 frogs
6x12	36 meters holds 72 frogs
7x11	36 meters holds 77 frogs
8x10	36 meters holds 80 frogs
9x9	36 meters holds 81 frogs

So a pen that is 9x9 uses 36 meters of wood and would hold 81 frogs.

Sean and Nabeel, age 10, Apprentice

1. 4 types of pens' dimensions are 1 x 17, 2 x 16, 3 x 15, and 4 x 14. 2. The amount of frog that can live in each are: 17, 56, 45, and 32

We found numbers that if multiplied by 2 and added together would be 36. Then we took a number from one and added it to the other. We did that again and again till we got all 4 dimensions.

Compared to the length of the main part (not counting the Extra) of William's solution, this pair's effort is quite a bit shorter. Notice, however, that Sean and Nabeel have tried to summarize what they did and I would praise them for that effort.

I would ask them to tell me more about the second part of the problem. I'd also ask how they knew that numbers multiplied by 2 and summed to 36 would give the pen's dimensions. I wonder if they might mention the word "perimeter."

Sarah, age 12, Practitioner

Pen A would be 12 by 6, Pen B would be 11 by 7, Pen C would be 10 by 8, and Pen D would be 14 by 4.

1. I got these perimeters by using guess and check. First I would pick a number for the length and then add it twice because in a rectangle there are two lengths and two widths. Next, I would subtract the sum from 36 to know how much I have left for the width. Lastly, I would divide it by two because width have to be equal. For example I picked 14 for the length, I added it twice and got 28. I subtracted 28 from 36 and got 8. So I had 8 meters left to use for the width. 8 divided by 2 equals 4. So I have 4 meters for each two sides. I now know both dimensions; 14 by 4.

2. Pen A which is a 12meter by 6meter rectangle, will hold 72 frogs. 12metersX6meters=72 square meters. Since each square meter holds one frog 72, squrae meters will hold 72 frogs.

Pen B which is an 11meter by 7meter recangle, will hold 77 frogs. 11metrsX7meters=77 square meters

Sarah has done a nice job with her solution. I might ask her to improve her Clarity score by working on her formatting.

I would suggest breaking up the one long paragraph that she's written to explain the first question. She might try the Extra.

Pen C which is a 10meter by 8meter rectangle, will have 80 frogs.
10metersX8metes=80 square meters

Pen D which is a 14meter by 4meter rectangle, will hold 56 frogs.
14metersX4meters=56 square meters

Matthew, age 0, Practitioner

1. One pen can have the width of 5 and a length of 13. One pen can have the width of 6 and a length of 12. One pen can have the width of 4 and a length of 14. One pen can have the width of 2 and a length of 16. 2. The first pen can hold 65 frogs. The second pen can hold 72 frogs. The third pen can hold 56 frogs. The fourth pen can hold 32 frogs. Extra: The pen with a length of 9 and a width of 9 has the largest area (81 square meters).

Frog Farming

1.
One pen can have the width of 5 and a length of 13.
One pen can have the width of 6 and a length of 12.
One pen can have the width of 4 and a length of 14.
One pen can have the width of 2 and a length of 16.

2.
The first pen can hold 65 frogs.
The second pen can hold 72 frogs.
The third pen can hold 56 frogs.
The fourth pen can hold 32 frogs.

The pen must be rectangular and include all 36 meters of fence.
This means that two times the length plus two times the width of the pen is equal to 36 meters. Since the pen is rectangular this means the length plus the width equals 18.
So to design a pen using the 36 meters of fence, you find all the combinations of two numbers that add up to 18.

To find out how many frogs fit in each pen, you need to find the area. The area of the pen is length times width.

Extra:
In order to find the pen that holds the maximum number of frogs, you must find the pen with the largest area. Out of all the possible combinations of length and width, the pen with a length of 9 and a width of 9 has the largest area (81 square meters). If you draw a table of all the possible combinations of length and width you can assure that you are correct.

Length	Width	Area
1	17	17
2	16	32
3	15	45
4	14	56
5	13	65
6	12	72
7	11	77
8	10	80
9	9	81

Matthew clearly formatted his explanation and at first it seems that he may not have explained his thinking enough but if you continue reading you can see he added those thoughts below his lists.

I would be tempted to ask him if he can see any patterns in his table. With the data so clearly displayed he might reflect and add in additional thoughts.

Scoring Rubric

A **problem-specific rubric** can be found linked from the problem to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work.

We hope these packets are useful in helping you make the most of Math Fundamentals Problems of the Week. Please let me know if you have ideas for making them more useful.

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