Math Fundamentals PoW Packet

Shopping at the School Store

Problem 4579 • https://www.nctm.org/pows/

Welcome

This packet contains a copy of the problem, the "answer check," our solutions, some teaching suggestions, and samples of the student work we received in November 2007. The text of the problem is included below. A print-friendly version is available using the "Print" link on the problem page.

Standards

In ***Shopping at the School Store*** students are asked to determine how much a pencil and an eraser cost at the school store. The **key concepts** are addition, multiplication, multiplies and algebraic reasoning.

If your state has adopted the Common Core State Standards, this alignment might be helpful.

Grade 3: Operations & Algebraic Thinking

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Grade 4: Operations & Algebraic Thinking

Gain familiarity with factors and multiples.

Grade 5: Operations & Algebraic Thinking

Analyze patterns and relationships.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
5. Use appropriate tools strategically.
7. Look for and make use of structure.

The Problem

Shopping at the School Store

Julio and David went to the school store. Julio bought 4 pencils and 3 erasers for 37¢. David paid 33¢ for 3 pencils and 4 erasers.

1. What is the cost of one pencil?
2. What is the cost of one eraser?

Explain how you solved the problem **and** how you know you are right.

Extra: Shonda met them coming out of the store. She went in and bought 12 of these items, for which she paid 44¢. How many pencils did she buy?

Answer Check

After students submit their solution, they can choose to "check" their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually **get** the answer) we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

A pencil costs 7¢. An eraser costs 3¢.

If your answer **doesn't** match ours,

- did you think about combining what Julio and David purchased and what they paid?

- did you make a table to keep track of what you tried?
- did you look for patterns?
- did you check your arithmetic?

If you used guess and check, did you tell . . .

- what numbers you tried?
- how you checked them?
- how you knew whether they worked or not?
- how you decided what to try next?
- about any patterns that helped you?

If any of those ideas help you, you might *revise* your answer, and then leave a comment that tells us what you did. If you're **still stuck**, leave a *comment* that tells us where you think you need help.

If your answer **does** match ours,

- is your explanation clear and complete?
- do you think there is another correct solution?
- did you try the Extra question?
- did you verify your answer (show how you know it works)?

Revise your work if you have any ideas to add. Otherwise leave us a *comment* that tells us how you think you did—you might answer one or more of the questions above.

Our Solutions

Method 1: Systematic Guess and Check, starting with maximum

Starting with Julio, I divided 37¢ by 4 to find the maximum cost per pencil: 37/4 = 9 r1

Pencils couldn't cost 9¢, because there wouldn't be enough money left for the 3 erasers.

I tried 8¢ for the cost of a pencil: 4 • 8¢ = 32¢

37¢ - 32¢ = 5¢ which represents the money left for erasers. It is not a multiple of 3. 8¢ per pencil won't work.

I tried 7¢ per pencil next: 4 • 7¢ = 28¢

37¢ - 28 ¢ = 9¢ which is a multiple of 39¢/3 = 3¢ the cost of one eraser

I know that these prices work for Julio:

4 pencils • 7¢ = 28¢
3 erasers • 3¢ = 9¢
28¢ + 9¢ = 37¢

I checked my answer by trying out these prices for David:

3 pencils • 7¢ = 21¢
4 erasers • 3¢ = 12¢
21¢ +12¢ = 33¢

Method 2 – Make a Table

We made lists of how much 4 pencils and 3 erasers would cost at different prices.

unit price	4 pencils	3 erasers
1¢	4¢	3¢
2¢	8¢	6¢
3¢	12¢	9¢
4¢	16¢	12¢
5¢	20¢	15¢
6¢	24¢	18¢
7¢	28¢	21¢
8¢	32¢	24¢
9¢	36¢	27¢
10¢		30¢
11¢		33¢

We looked for combinations that would total 37¢, what Julio paid for 4 pencils and 3 erasers. Any price for pencils will result in an even number, so we knew the cost of the 3 erasers had to be an odd number in order to make a sum of 37¢. We only needed to consider the odd prices for erasers. We found these:

9 + 28 = 37 [3 erasers @ 3¢, 4 pencils @ 7¢]
21 + 16 = 37 [3 erasers @ 7¢, 4 pencils @ 4¢]
33 + 4 = 37 [3 erasers @ 11¢, 4 pencils @ 1¢]

We knew any of these prices would work for Julio's purchase, so we tried these prices with David's.

4 erasers @ 3¢ and 3 pencils @ 7¢ = 12¢ + 21¢ = 33¢ It works!
4 erasers @ 7 ¢ and 3 pencils @ 4 ¢ = 28¢ + 12 ¢ = 40¢ No.
4 erasers @ 11¢ and 3 pencils @ 1¢ = 44¢ + 3 ¢ = 47¢ No.

Now we know pencils cost 7¢ and erasers cost 3¢.

Extra: We began testing numbers of pencils (multiples of 7¢) to find which could combine with a number of erasers (multiple of 3¢) to total 12 items and 44¢.

	pencil cost	**difference (44¢ - pencil cost)**	**multiple of 3?**
1	7¢	37¢	no
2	14¢	30¢	yes

Since 30¢ is the cost of 10 erasers, and adding of 2 pencils at 7¢ each (14¢) would bring the total to12 items and 44¢, we know Shonda purchased 2 pencils and 10 erasers.

Method 3 – Direct Approach, using combined purchases

If Julio and David combined their purchases, they would have 7 pencils and 7 erasers for a total of 70¢ spent. That means that 10¢ will buy one pencil and one eraser.

It would cost 30¢ for 3 of each. Since 4 pencils and 3 erasers costs 37¢, a pencil must cost 7¢.

37¢ - 30¢ = 7¢

It will cost 28¢ for 4 pencils at 7¢ each. The difference, 37¢ - 28¢ = 9¢, is the cost of 3 erasers, so they must cost 3¢ each.

OR

4 pencils and 3 erasers cost 37¢; 3 pencils and 4 erasers cost 33¢. Trading a pencil for an eraser saves 4¢. A pencil costs 4¢ more than an eraser. One of each costs 10¢, so a pencil costs 7¢ and an eraser costs 3¢.

Method 4 - Algebra

I made two equations to represent each boy's purchase, using p for pencils and e for erasers:

4p + 3e = 37
3p + 4e = 33

I multiplied all terms of the first equation by -3, and the second equation by 4, to eliminate the pencils:

-12p - 9e = -111
12p + 16e = 132
7e = 21
e = 3

Substituting in the first equation: 4p + 9 = 37

4p = 28
p = 7

Pencils cost 7¢ and erasers cost 3¢.

Check:

(4 • 7) + (3 • 3) = 37
(3 • 7) + (4 • 3) = 33

Extra: If all 12 items were erasers, they would only cost 36¢. Trading an eraser for a pencil increases the total cost by 4¢: 11 erasers (33¢) + 1 pencil (7¢) = 40¢. Another trade increased the total cost to 44¢: 10 erasers (30¢) + 2 pencils (14¢) = 44¢. I know that Shonda bought 2 pencils.

Teaching Suggestions

Solvers of ***Shopping at the School Store*** need to find the cost of one pencil and of one eraser in order to satisfy the given costs of two different groupings of them. It can be modeled with manipulatives, with real pencils and erasers, or with a diagram. Children who use a form of guess and check should be encouraged to keep a good record of their trials and to report the details of their process. Students can apply algebraic reasoning to sort out the relationships among the numbers in the problem, even if they are not yet ready for formal algebraic procedures. Students using *Math in Context* might use a Combination Chart.

The questions in the Answer Check, above, might serve as useful prompts to help students make progress. Encourage students to use a strategy that works for them. As you can see from many methods we used in Our Solutions, above, your students might approach this problem in a number of different ways.

Sample Student Solutions

focus on **Strategy**

In the solutions below, we've provided the scores the students would have received in the **Strategy** category of our scoring rubric. Our comments focus on what we feel is the area in which they need the most improvement.

Novice	Apprentice	Practitioner	Expert
Has no ideas that will lead them toward a successful solution or shows no evidence of strategy.	Uses a strategy that uses luck instead of skill, or doesn't provide enough detail to determine whether it was luck or skill.	Uses a strategy that relies on skill, not luck, which might include: • thorough noticing and wondering • guess and check • make a list, chart, or table • logical reasoning	Does one or more of these: Uses two different strategies. Uses a good Extra strategy. Uses an unusual or sophisticated strategy, e.g., effective and appropriate use of technology or algebra.

James

age 12

Strategy **Novice**

Erasers are five cents and pencils are three cents .

I picked amounts of cents, that when multiplied by the number of supplies would equal 33 and 37 cents. I found that five and three multiplied by certain amounts of supplies, gets 33 and 37.

If James had included some of his "picks" (guesses) I might consider this an Apprentice in strategy because I'd have some sense of if it really worked or not. With little explanation I decided to err on the side of caution and mark it as a Novice since there just isn't enough to decide.

I would ask James for some details and, perhaps, suggest he use a table, list or chart to keep track of his "picks."

Moriah
age 8

Strategy
Novice

I used a chart.

Julio had 4 pencils 7 cents 3 ereasers for 9 cents.

Moriah's idea of using a chart is a good strategy but I can't quite give her credit for it since she's not given me any description of what it looked like or tried including it in any way.

I would ask her to tell me about her chart. What information did she keep track of in it?

Jack and Ryan
age 9

Strategy
Apprentice

The pencil costs three cents and the eraser costs 2 cents

We did 37 divided by 7 = 5, then we did 5 divided by 3 = 2, so erasers cost 2 cents and pencils cost three cents

Finding the "unit cost" might be one way to start this problem but I'm not convinced that Jack or Ryan used the information they got by dividing to think more about the problem.

I might ask them if there were any remainders when they divided and what that meant. I might also ask if they checked their answers by thinking about what Julio or David paid for their erasers and pencils.

A
age 11

Strategy
Apprentice

The price for a pencil is 7 cents. The price for an eraser is 3 cents.

I started by finding all of the posible combinations that would work for Julio's purcase and I found that these work. And no other combination did.

1 pencil = 4cents and 1 eraser = 7cents 4+4+4+4+7+7+7=37
or
1 pencil = 7cents and 1 eraser = 3cents 7+7+7+7+3+3+3=37

then I tried to see which one worked on david's purchase and only the second one worked.
7+7+7+3+3+3+3=33
This is what david's purchase would look like if I used the first meathod
4+4+4+7+7+7+7=40

A has referred to the idea that all of the possible combinations were checked but it's hard to know what strategy was used to accomplish that work.
I might ask A to tell me about that work including some of the first guesses used. How did those guesses help find the better possibilities.

Cassie
age 10

Strategy
Practitioner

The answer for this math forum problem is pencils are 7 cents and erasers are 3 cents. Extra: 2 pencils

I guessed that each pencil costs 6 cents but I was wrong. So I used the same strategy but saying that each pencil costs 7 cents.

4-pencils
x7-amount of money for 1 pencil
28-amount of money for 4 pencils

Cassie has written a clear and complete explanation showing how her guess and check strategy worked to help her find the solution.

37 cents-amount of money paid by Julio
-28 cents-amount for all the pencils
9-total amount of money for erasers

I subtracted 28 cents from the total amount of money spent which was 37 cents. This way I could find the answer of how much money was spent on erasers by Julio. Since the total amount for erasers is 9, I have to divide that by 3.

9 divided into 3 is 3.

Right now I think that an eraser costs 3 cents and a pencil costs 7 cents. So now I have to check with the amount of money paid by David.

3cents- money for erasers
x4- erasers
12

7 cents-money for pencils
x3-pencils
21

Now I add them up. 12+21=33 cents

Since David's caluclations added up the answer is 3 cents for erasers and 7 cents for one pencil.

Extra

I estimated that Shonda bought 4 pencils but when I added up the erasers it didn't add up right. Then I estimated 3 pencils but that didn't work either. So I tryed 2 pencils:

7 cents- amount of money for one pencil
x2- pencils
14

44 cents- total amount of money spent by Shonda
-14- total money spent on pencils
30

Now I divide 3 cents (erasers) into the amount spent on erasers (30 cents).

3 divided into 30 equals 10.

Shonda spent 14 cents on 2 pencils and 30 cents on 10 erasers.

Jimmy
age 10

Strategy
Practitioner

At the school store, pencils are $00.07 and erasers are $00.03.

Shopping at the School Store
Parts I and II
P=Pencil E=Eraser
For part I (question 1.), I made guess and check list like this:

Julio-37 cents
x4-P | x3-E
---
5 | 3 = 29
---
6 | 4 = 48
---
7 | 3 = 37

Jimmy has organized his work using a table. He's done a nice job explaining how he decided what to include in his table. I might suggest that he try the Extra.

What is above is the guess and check table I drew is Julio's purchase. The reason for the x4 | x3 is that to find what they paid you must multiply the number of items purchased by its price to find the total price of every item. What is being guessed is the price of each individual item. The "formula" I used for this problem would be (pencil price x number of pencils purchased) + (eraser price x number of erasers purchased) = overall price of items for Julio. The numbers to Julio's equation are (7 x 4) + (3 x 3) = 37 cents. 7 and 3 are the two number guessed, and the numbers are prices. So, that is how I know my answer is right. I did the same thing with David.

Jonathan
age 9

Strategy
Expert

SQ1: The cost of one pencil is 7 cents and the cost of an eraser is 3 cents. extra: Shonda bought 2 pencils.

Shopping at School Store
Q1: A pencil at the school store costs X cents
An eraser costs Y cents
4X + 3Y = 37---- (1)
3X + 4Y = 33----- (2)
(1)+ (2) = 7X + 7Y=70
X + Y = 10 ---- (3)
(1)- (2) = X - Y= 4 --- (4)
(3) + (4) =(X + Y) + (X – Y) =14
2X = 14
X = 7
Y = 3
A pencil costs 7 cents, an eraser costs 3 cents
I checked my answer by:
Julio: 4 pencils + 3 erasers = 4x7+3x3=28+9= 37
David: 3 pencils + 4 erasers = 3x7+4x3= 21+12=33

Extra:
Shonda bought X pencils and Y erasers
X + Y = 12 => X = 12 - Y
7X + 3Y = 44
7x (12 – Y) + 3Y = 44
84-7Y+ 3Y = 44
-4Y = 44-84
-4Y =-40
Y=10
X = 2
Shonda bought 2 pencils.
I checked my answer by:
7X + 3Y = 44
7 x 2+3x10=14+30=44

Jonathan has used an algebraic strategy for both the main problem and also the Extra. In both cases he has checked his work. Great job!

Scoring Rubric

A **problem-specific rubric** can be found linked from the problem to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work.

We hope these packets are useful in helping you make the most of Math Fundamentals Problems of the Week. Please let me know if you have ideas for making them more useful.

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