NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

# Problem of the Week Teacher Packet

# Counting Chicken Wings

At Annie's Home-Cooked Chicken Wings Restaurant, chicken wings are served by the bucket. The Biggest Bucket O' Wings is really big! Let's figure out how many wings are in it.


If they're removed two at a time, one wing will be left. If they're removed three at a time, two wings will remain. If they're removed four, five, or six at a time, then three, four, and five wings, respectively, will remain. If they're taken out seven at a time, no wings will be left over.

What's the smallest possible number of wings that could be in the bucket? How do you know?

## Answer Check

After students submit their solution, they can choose to "check" their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually get the answer) we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

*The smallest possible number of wings that could have been in the bucket is 119.*

If your answer **doesn't** match ours,

- did you realize that the number of wings in the bucket is odd?
- did you consider that the number of wings in the bucket must be a multiple of a specific number?What is it?

If any of those ideas help you, you might revise your answer, and then leave a comment that tells us what you did. If you're still stuck, leave a comment that tells us where you think you need help.

If your answer **does** match ours,

- have you clearly shown and explained the work you did, so that a fellow student would understand?
- did you make any mistakes along the way? If so, how did you find and fix them?
- did you show how you know your answer is right?

Revise your work if you have any ideas to add. Otherwise leave us a comment that tells us how you think you did—you might answer one or more of the questions above.

## Our Solutions

**Method 1: Exhaustive List**

I made a chart of numbers and as I checked each number to see if it worked I marked off the numbers that didn't work. I started with 1 but that was kind of silly because 1, 2, 3, 4, 5, and 6 can't be divided by 7 evenly. So, I quickly marked those off my chart.

7 works for some of the numbers:

7/2 = 3 R 1, but 7/3 = 2 R 1 and so that isn't right.

I kept going, checking each number until I got one part that didn't work. As I was working, I figured out that the number would have to be odd because none of the even numbers could be divided by 2 and still have a remainder of 1. Once I figured that out I crossed out all of the even numbers. Then I started thinking that if 7 went into the number with no remainder, the number would have to be a multiple of 7. I crossed out all of the numbers on my chart that weren't multiples of 7.

That made my work a little easier. When I finally got to 119, it worked for all of the numbers!

119/2 = 59, remainder 1 , 119/3 = 39, remainder 2, 119/4 = 29, remainder 3 , 119/5 = 23, remainder 4 ,
119/6 = 19, remainder 5, 119/7 = 17, remainder 0

**Method 2: Multiples of 7**

Since the problem says that if you take the wings out seven at a time there won't be any wings left over, I know that the number of wings will be a multiple of 7. So, I started thinking of the multiples of 7:

7 works for 2 because 7/2 is 3 with a remainder of 1 but it doesn't work for 3 because 7/3 is 2 with a remainder of 1.

Next, I tried 14 but it didn't work for 2 because 14/2 is 7 with no remainder.

I tried 21 but it didn't work for 3 because 21/3 is 7 with no remainder.

I tried 28 but it didn't work for 2 because 28/2 is 14 with no remainder.

I tried 35 but it didn't work for 5 because 35/5 is 7 with no remainder.

I tried 42 but it didn't work for 2 because 42/2 is 21 with no remainder.

I tried 49 but it didn't work for 3 because 49/3 is 16 with a remainder of 1.

I tried 56 but it didn't work for 2 because 56/2 is 18 with no remainder.

I tried 63 but it didn't work for 3 because 63/3 is 21 with no remainder.

I tried 70 but it didn't work for 2 because 70/2 is 35 with no remainder.

I tried 77 but it didn't work for 2 because 77/4 is 19 with a remainder of 1.

I tried 84 but it didn't work for 2 because 84/2 is 42 with no remainder.

I tried 91 but it didn't work for 2 because 91/3 is 30 with a remainder of 1.

I tried 98 but it didn't work for 2 because 98/2 is 49 with no remainder.

I tried 105 but it didn't work for 5 because 105/5 is 21 with no remainder.

I tried 112 but it didn't work for 2 because 112/2 is 56 with no remainder.

I tried 119 and it worked! Whew, I found the number. It's 119.

**Method 3: Intelligent Elimination**

I know that the number is a multiple of 7. I also know that it ends in 4 or 9 because it is 4 more than a multiple of 5 (and multiples of 5 end in 0 or 5). But it can't end in 4 because it has to be odd, otherwise there won't be a remainder when you divide it by 2.

So it is a multiple of 7 that ends in 9. There aren't many of those! I found 49, 119, and 189. Then I checked with the other rules to see which one works.

If I divide 49 by 3, I don't get a remainder of 2, I only get a remainder of 1. So that can't be it.

I tested 119 and it worked with everything, so that is the smallest possible number of wings that could be in the bucket. I didn't have to test the 189 since I already found the answer.

**Method 4: Divisibility to the Extreme**

Let N be the number of wings.

Since there was 1 left when dividing by 2, N must be one less than a multiple of 2. Since there were 2 left when dividing by 3, N must be one less than a multiple of 3. Since there were 3 left when dividing by 4, N must be one less than a multiple of 4. Since there were 4 left when dividing by 5, N must be one less than a multiple of 5. Since there were 5 left when dividing by 6, N must be one less than a multiple of 6. Since it is 1 less than multiples of all those numbers, it is also 1 less than a multiple of 60, which is the least common multiple of all of those numbers. We also know N is a multiple of 7.

60 – 1 = 59, not a multiple of 7.

120 – 1 = 119. Yes! A multiple of 7.

## Standards

If your state has adopted the Common Core State Standards, you might find the following alignments helpful.

*Grade 6: The Number System*

Apply and extend previous understandings of multiplication and division.

*Mathematical Practices*

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.

## Teaching Suggestions

In the sample solutions, you'll see students who forgot to take into account all of the conditions set out in the problem, or didn't entirely understand the information in the problem. There were a variety of ways that the facts given in the problem could be used. Many students started with the fact that the number of wings must be a multiple of 7 because when you took out the wings 7 at a time there would be nothing left in the bucket.

Some prompts or questions you might use with students include:

- Using the given information in the problem, is it possible to tell whether the number of wings in the bucket is even or odd?
- The number of wings in the bucket must be a multiple of what number?
- If a number is divisible by 5, the units digit is either 0 or 5. Given this fact, when 4 remains after a number is divided by 5, what can we say about the units digit of that number?

Sometimes having students try a simpler problem can help:

**Simpler Problem 1:** You have an amount of candy that you are dividing between kids. When you have 2 kids and divide the candy so that each kid gets an equal amount, there is 1 piece of candy left over. When you have 3 kids and divide the candy so that each kid gets an equal amount, there is no candy left over.

What do you know about the amount of candy that you were dividing?

**Simpler Problem 2:** You have a sheet cake that you are dividing so that each person will get an equal amount. You cut the cake so that:

There would be 1 piece left over if it were for 2 people.
There would be 1 piece left over if it were for 3 people.
There would be 1 piece left over if it were for 4 people.
It would work perfectly for 5 people to each get the same amount.

How many pieces were there?

 

You might also provide manipulatives so that they could act out some of the problem (though you might not want everyone to have 119 of anything!).

Encourage students to use a strategy that works for them. You can see from the methods that we have thought to use for this problem that there is more than one way to approach it. And keep in mind that we may not have thought of them all!

## Sample Student Solutions – Focus on *Strategy*

In the solutions below, I've provided scores the students would have received in the **Strategy** category of our scoring rubric. My comments focus on areas in which they seem to need the most improvement.

| Novice | Apprentice | Practitioner | Expert |
|---|---|---|---|
| Has no ideas that will lead them toward a successful solution or shows no evidence of strategy. | Uses a strategy that uses luck instead of skill, or doesn't provide enough detail to determine whether it was luck or skill. | Uses a strategy that relies on skill, not luck, which might include:<br>• using two or fewer of the clues to narrow things down before listing numbers, i.e. lists all numbers<br>• uses the fact that the answer must be a multiple of 7<br>• uses the fact that it must be an odd multiple of seven and then makes a starting list. | Uses at least three divisibility rules to narrow things down before they list any numbers (i.e. an odd multiple of 7 that ends in 4). This includes the strategy of using the LCM of 2, 3, 4, 5, and 6. |

**Julie, age 14, Novice**

I think the number is 77!!!!

I do not know how I came up to this number because im just guessing its 50 50 chance on getting it write.

*Even though Julie claims that she is guessing, I think she realizes that the answer needs to be a multiple of 7. I might point out that the problem says that if you take two wings out at a time that there will be one left over and there are other conditions for 3, 4, 5, and 6.*

**Flora, age 11, Apprentice**

The smallest number of wings possible is 35.

I figured that there were 35 wings in the bucket because first of all I used the guess and test strategy to find the answer. I first looked at my facts: the number of wings in the bucket were divisible by 7. So I then thought that it had to be an odd number because when divded by 2, there is 1 piece left over. Therefore, I guessed 35, which is divisible by 7, and looked to see whether all the clues connectedto 35. So, that's why my answer is 35.

*I like how Flora used "guess and test" to describe her strategy. (We like to add one word and call it "guess, test, and revise.")*

*Flora has a good start using the divisible by 7, odd number, and dividing by 2 ideas. I would remind her that there are a few other rules, and ask her to see what happens when she divides her answer by 3, 4, 5, and 6.*

**Andrea, age 12, Apprentice**

It whould be 1 chicken wing because your would take out 2 at a time 1 wing would be left

because it syas it right in the problem thta if you take out 2 of the wings there would be 1 left it is a trick

*Andrea would have a good idea, if she were right that we were tricking her! Perhaps once we assure her that it isn't a trick, she'll be able to look back at the clues in the problem.*

**Kathyrn, age 12, Practitioner**

My answer is that the smallest possible number of wings in the bucket is 119 wings.

If when you take out the wings by 7, there are none left, then the amount in the bucket has to be divisable by 7.

If there is one wing left when you take out wings two at at time,then the number of wings in the bucket must be an odd number.

If there is a remainder of wings left when you take out wings two,three,four,five or six at a time then the number can't be divisable by any of these numbers.

We are told that if the wings are removed three at a time two will remain and if they're removed four,five or six at a time, then three four and five wings respectively would remain.

So I then looked for the smallest multiple of seven that would satisfy all the conditions provided by the problem.

Seven doesn't work because if we take out the wings three at a time two must remain. Fourteen doesn't work because it is divisable by two. Twenty one doesn't work because it is divisable by three.

Twenty eight, thirty five, and forty two do not work because they are divisable by four ,five and six and in any event twenty eight and forty two are even numbers.

Forty nine doesn't work because when you take out wings in threes there is only one left. Fifty-six is even and 63 is divisable by three. Seventy seven doesn't work because when you take out wings in fives,two are left. Eighty four is even.

Ninety one doesn't work because only one wing is left when you take out wings in threes. Ninety eight is even. One hundred and five is divisable by three. One hundred and twelve is even. One hundred nineteen is the first multiple of seven that satifies all the conditions.

*Kathyrn has a sound method that uses all of the clues given in the problem. First she knows that the answer has to be divisible by 7 and then she decides that the answer must be odd.*

*Using those two clues she methodically starts with 7 and continues testing until she gets to 119.*

*I might ask her if she can eliminate some of the numbers she tested without having to "test" them (for example all of the even multiples of seven).*

**Kelvin and Brittany, age 13, Practitioner**

The smallest number of wings in the bucket is 119. We worked it out by drawing a chart with the information you provided for us.

| CHICKEN WINGS REMOVED | CHICKEN WINGS REMAINED |
|---|---|
| 2 | 1 |
| 3 | 2 |
| 4 | 3 |
| 5 | 4 |
| 6 | 5 |
| 7 | 0 |

*Kelvin and Brittany made a chart to help them think about the information provided in the problem. One thing I might ask them to include that would make their solution a little more complete would be a list of the multiples of seven that they tested before getting the answer of 119.*

The fact that when the wings are removed 2 at a time, there was 1 left, told us that that the number would be odd. The fact that when 7 were removed, 0 were left, told us that the number was a multiple of 7. We divided the multiples of 7 by the numbers of chicken wings removed to see if the answer would satisfy the number of wings removed and the wings left over. When we got up to 119, we found out that when 119 was divided by 2, 1 was left. When 119 was divided by 3, 2 were left. When 119 was divided by 4, 3 were left. When 119 was divided by 5, 4 were left. When 119 was divided by 6, 5 were left. When 119 was divided by 7, 0 wings were left.

**Stephen, age 12, Practitioner**

The answer to "Counting Chicken Wings" is 119.

FIrst I wrote out all numbers betweem 1 and 200. Then I crossed out all numbers that were even because they are divisible by two and therefore would not work. I did the same with three, four, and five until I was left with mostly prime numbers. THen I crossed out all numbers that weren't divisible by seven because seven fits into the number of chicken wings evenly. Then I was left with the numbers 7, 49, 91, 119, 133, 147, and 161. I then found out that 7 would be too small. Forty-nine would not leave two chicken wings after taking out three at a time. Ninety-one wouldn't work it too wouldn't leave two chicken wings after taking out three at a time. Once I got to 119, I found out that this was the correct answer by subtracting 1 (118) and then dividing that difference by two. Then I subtracted two (117) and divided by three. These equations all worked out evenly and I did the same for four and five. I took one less then the number, subtracted it from 119, and then took the original number and divided it by the difference. This eventually all worked out evenly and that's how I came to the answer of 119.

*Stephen has a sound strategy although it seems like it might take some time to write all of the numbers from 1 to 200. I might ask him if he noticed anything about what numbers he was crossing out. I would wonder if he did a similar problem if he would write the same numbers or if he could eliminate some at the start.*

**Katie, age 11, Practitioner**

The smallest amount of chicken wings that could be in the bucket it 119.

1. I listed all multiples of 7 up to 150

7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84. 91, 98, 105, 112, 119, 133, 147

2. I crossed out all the even numbers because if you take 2 out and there is one left it has to be an odd number.

Remaining numbers: 7,21,35,49,63,77,91,105,119,133,147

3. I then crossed out all multiples of three because if you take out three chicken wings and there are two remaining it means that 3 cannot go evenly into that number.

Remaining numbers:7,35,77,91,119,133

4. I then crossed out all the multiples of 5.

Remaining numbers:7,77,91,119,133

5. Now I go back and see if when divided by 2,3,4,5,6 you get the correct remainder.

7/2=3 remainder 1
7/3=2 remainder 1
This rules out 7

77/2=38 remainder 1
77/3=25 remainder 2
77/4=76 remainder 1
This rules oout 77

*Katie's strategy is similar to Stephen's except she made some decisions before making her list. She was methodical in listing all of her calculations. That might seem like too much to include, but it could be very helpful for a classmate to use to understand what she did.*

91/2=45 remainder 1
91/3=20 reminder 1
This rules out 91

119/2=59 remainder 1
119/3=39 remainder 2
119/4=29 remiander 3
119/5=23 remainder 4
119/6=19 remainder 5
119/7=17 no remainder

We have found our answer of 119!

**Natalie, age 9, Practitioner**

The least number of wings that can go in the bucket is 119.

Data: x - the number of wings in the bucket

Calculations: From the data in the question, I can make 6 equations:

1. 2a + 1 = x
2. 3b + 2 = x
3. 4c + 3 = x
4. 5d + 4 = x
5. 6e + 5 = x
6. 7f = x

From equation 1 and 6, I can derive that x is odd and a multiple of 7, so possible values of x are:

x = 7, 21, 35, 49, 63, 77, 91, 105, 119, 133, 147, ...

Next, I substitute the values for x, and if all equations will have integers as outcomes, that is the least possible value for x:

For x = 7, a = (7-1)/2 = 3

b = (7-2)/3 = 5/3 , which is not an integer

I do the same for the rest of the equations and find that the least integer possible to be x is 119. So 119 chicken wings are in the bucket.

*Natalie uses a little algebraic notation to help her think about the problem. I wonder if she actually wrote all of the substitutions before she found that 119 worked or if she was able to do that mentally once she wrote out the first example.*

**Aditya, age 11, Expert**

The lowest possible number of wings in the bucket is 119 (too bad I'm a vegetarian)

The reason that this is true is as following...

If there will be N left over when removing in intervals of (N-1) (I'll call it D) then it is like saying...

If P is the dividend and Q is the quotient...

P / D = Q r N

These facts clearly help state that I should look for the lowest # with these requirements:

The amount has to be ODD (dividing by two)

The amount has to be a multiple of seven (dividing by 7)

The amount has to end in four or nine BUT only nine is possible (dividing by 2, 5)

*Aditya's solution points out some nice number theory ideas. Although she has an expert in strategy score, I would like to understand more about why she made some of the decisions that she noted. I'm not sure that there is enough explanation included for another student to clearly follow what she is saying.*

With these few facts, the # gets narrowed down to 119.

I first tried 49, but it didn't fit 3 or 6.

Then I added 70 to keep the nine in place.

I checked and 119 worked...

**Tim, age 13, Expert**

The smallest possible number of wings that could be in the bucket would be 119.

First, let the smallest number of chicken wings be x.

If we take two, three, four, five, six at a time, there will be one, two, three, four and five left. So if we had one more, it would be the LCM for 2,3,4,5,6 and remains 1 when divides by 7, which is also x+1.

So: x+1=LCM{2,3,4,5,6} and X divisible by 7
x+1=LCM(2)(3)(2*2)(5)(2*3)
x+1=2*2*3*5
x+1=60

So the multiple of 60's would divisible by 2,3,4,5,6. then x could by 59,119,179... and 119 is the smallest number which can be divided by 7.

*Tim went a step farther than most of the students and realized that if he used the least common multiple, he could quickly determine the correct number.*

*Besides using an expert strategy, he has formatted his solution so that it's clear to read and also included enough steps to be complete.*

## Scoring Rubric

A **problem-specific rubric** can be found linked from the problem to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work.

We hope these packets are useful in helping you make the most of the Pre-Algebra Problems of the Week. Please let me know if you have ideas for making them more useful.

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