Pre-Algebra PoW Packet

Forming Triangles from a Folding Ruler

Problem 3416 • https://www.nctm.org/pows/

Welcome

This packet contains a copy of the problem, the "answer check," our solutions, some teaching suggestions, and samples of the student work we received in May 2005. The text of the problem is included below. A print-friendly version is available using the "Print" link on the problem page.

Standards

In ***Forming Triangles from a Folding Ruler*** students are asked to describe the three triangles that can be made with the full length of a 6-foot long folding wooden. The **key concept** is the Triangle Inequality Property and knowing the different types of triangles as classified by edgelength. At this level students may not formally mention the Triangle Inequality Property but from their explanations it will be informally noted.

If your state has adopted the Common Core State Standards, this alignment might be helpful:

Grade 7: Geometry

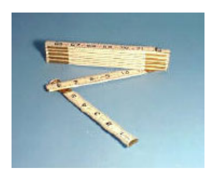
> 7.G.2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The Problem

Folding Triangles from a Folding Ruler



A folding ruler is a wooden ruler made of straight hinged sections that you can fold or unfold depending on how much ruler you need. The one that I have is 6 feet long and is split into 12 sections of 6 inches each.

Imagine bending the ruler to form the shape of a triangle.

- What are the dimensions of the three different triangles that each have a perimeter of 72 inches?
- Which triangle is scalene? isosceles? equilateral?

Answer Check

After students submit their solution, they can choose to "check" their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually **get** the answer) we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

> The equilateral triangle is 4 sections by 4 sections by 4 sections. It measures 24" by 24" by 24".
> The isosceles triangle is 5 sections by 5 sections by 2 sections. It measures 30" by 30" by 12".
> The scalene triangle is 5 sections by 4 sections by 3 sections. It measures 30" by 24" by 18".

If your answer **doesn't** match ours,

- did you remember that you only have 12 sections of the ruler to work with?
- did you remember that each of them is 6 inches in length?
- did you remember the characteristics of the different types of triangles?

If any of those ideas help you, you might *revise* your answer, and then leave a comment that tells us what you did. If you're **still stuck**, leave a *comment* that tells us where you think you need help.

If your answer **does** match ours,

- have you clearly shown and explained the work you did?
- are you confident that you could solve another problem like this successfully?
- did you make any mistakes along the way? If so, how did you find and fix them?
- are there any hints that you would give another student?

Revise your work if you have any ideas to add. Otherwise leave us a *comment* that tells us how you think you did—you might answer one or more of the questions above.

Our Solutions

Method 1: Plan and Reflect

I noticed and wondered in terms of quantities and relationships.

Quantities:
The wooden ruler is 6 feet long.
The ruler can be folded.
Each folded section is 6 inches long.
There are 12 sections in all.
The perimeter of the triangle is 72 inches.

Relationships:
There are 12 inches in 1 foot.
A triangle has three sides.
When the ruler is bent it can form a triangle. Since the perimeter of the triangle is 72 and 12 x 6 is 72, this just means there's no overlapping.
The kinds of triangles mentioned are scalene, isosceles, and equilateral. An equilateral triangle has 3 equal sides. An isosceles triangle has 2 of its 3 sides equal and a scalene triangle has no equal sides.

Make a Table
I can consider all of the possible ways to bend the folding ruler and see if the combinations make a triangle. Once I know the combinations of sections I could figure out the lengths and also what kind of triangle it is.

Make a Mathematical Model
I can look around the house to see if we have one of those kinds of rulers. If we don't I could use pencils or wooden sticks or toothpicks to see how it might work. I'll need to make sure all 12 things are about equal in length.

My Plan:
Make a table.
Consider the possible combinations of sections.
Decide if a triangle can be made knowing that when I add any of the two sides it should be longer than the third side. If it isn't then when you bend the pieces they won't connect.
Find the lengths.
Decide what kind of triangle each is.

# of sections making Side 1	# of sections making Side 2	# of sections making Side 3	?	Notes
1	1	10	no	actually none of the triangles will have a side of 1 – none of them make triangles
2	4	6	no	adding side 1 and 2 gives 6 and that equals side 3 – no triangle – to make it work sides 2 and 3 have to be equal
2	5	5	yes	I continue making sure that the sum of any two is always bigger than the third
3	5	4	yes	
3	4	5	yes	
4	4	4	yes	

4	3	5	yes	
4	5	3	yes	
5	5	2	yes	
5	4	3	yes	
5	3	4	yes	
5	2	5	yes	

As I considered the different combinations I could see that some were repeated. What I have are just three possibilities:

4, 4, 4 or 24 inches by 24 inches by 24 inches. That's an equilateral triangle.

2, 5, 5 or 12 inches by 30 inches by 30 inches. That's an isosceles triangle.

3, 4, 5 or 18 inches by 24 inches by 30 inches. That's a scalene triangle.

Method 2: Working from the Number of Sections

I have 12 sections of the ruler to work with. I know that I can make an equilateral triangle with an edgelength of 4 sections because when I divide 12 by 3 I get exactly 4. To find the lengths of the sides of this triangle, I multiply the edgelength by 6, since each section is 6 inches long. The edgelength will be 24 inches.

If I make one side shorter by 1 section, another side has to get longer by 1 section. So the sides are 3, 4, and 5 sections long. Since all the edges are different lengths, this is a scalene triangle.

If I make the 3-section side shorter again by 1, another side has to be longer by 1. I can make the 5 side into a 6. But then it doesn't work as a triangle - any two sides of a triangle have to add up to more than the third side or you don't get a triangle. Here we have 2 and 4, and a 6. And the 2 and 4 equal 6. So no go.

But I can make the 4-section side one longer to get 2-5-5. This works as a triangle. The edges have lengths of 12 inches, 30 inches, and 30 inches. It is also isosceles, since two of the sides are the same length.

The problem says we have to find three, and we have, and so we are finished.

Method 3: Working from the Types of Triangles

To make an equilateral triangle, that means all the sides are the same length. I can divide 12 by 3 to get 4. Each side is 4 sections long.

To make an isosceles triangle, two of the sides have to be the same. First I used 3 and 3. This leaves 6. But that won't work because the 3 and 3 together aren't longer than the 6, and it won't make a triangle. I can't use 4 and 4 since we used that already. I tried 5 and 5. That leaves 2, and that works okay. The isosceles is 5-5-2.

Now I have to make a scalene triangle, which means that all of the sides have to be different. First I tried using 2 and 3, but that leaves 7 left over, and that is too long. Next I tried 3 and 4. That leaves 5, and that will work. The scalene triangle is 3-4-5.

Method 4: Making an Exhaustive List

One way to solve this is to make a list of all of the possible combinations of edgelengths from the 12 sections. Such a list might start:

1-1-10
1-2-9
1-3-8
1-4-7
1-5-6

We stop there with 1 as the first choice because the next one (1-6-5) is just a repeat of the previous combination. The list then continues with 2 as the first choice. To avoid repeats, we don't need to consider any second edgelength less than the first, or any third edgelength less than the second. So we continue the list:

1-1-10	2-2-8	3-3-6	4-4-4
1-2-9	2-3-7	3-4-5	
1-3-8	2-4-6		

1-4-7 2-5-5
1-5-6

That gives us 12 possibilities. But wait! The problem tells us that there are only three triangles that work. We need to figure out which ones will actually form triangles. Any two sides of a triangle together have to be longer than the third side. Otherwise they won't reach.

The Triangle Inequality Property can be stated like this. For three segments of length a, b, and c, the three can form a triangle if and only if the following conditions are true:

$a + b > c$
$b + c > a$
$a + c > b$

Now I can check each of the possibilities above. The first column, with 1 as the first length, is easy. None of them will work, since the sum of the first two sides is never greater than the third side.

In the second column, the first three don't work (because of the sum of the first two sides), but the fourth does. 2-5-5 works because $2 + 7 > 5$ (and $5 + 5 > 2$).

In the third column, the first one doesn't work because $3 + 3 = 6$, instead of being greater than 6. The second one does work, though. And the one in the fourth column works fine.

These are the three possibilities:

2 – 5 – 5
3 – 4 – 5
4 – 4 – 4

To find the actual edgelengths, I'll multiply all of the numbers by 6 inches.

12" by 30" by 30"
18" by 24" by 30"
24" by 24" by 24"

That's the answer to Question 1. For Question 2, we have to figure out which triangle is scalene, isosceles, and equilateral.

Equilateral means that all of the sides are equal, so that would be 24-24-24. Isosceles means that at least two of the sides are equal, so that would be 12-30-30. (24-24-24 is also isosceles, but the problem asks for one of each.) The scalene triangle needs to have all three sides different, so that's the 18-24-30.

Method 5: Algebra

1. The folding scale is divided into 12 equal segments. Therefore it is better if we take the number of segments as the unit of our length.

Let the sides of the triangles be x, y and z.
Therefore $x + y + z = 12$..........................(i)

In a triangle, the sum of any two sides is always greater than the third side. Therefore,
$(x + y) > z$....................(ii)
$(x + z) > y$....................(iii)
$(y + z) > x$....................(iv)

If we substitute the value of $(x + y)$ from (i) into (ii), we get $12 – z > z$ or $z < 6$. Similarly we get $x < 6$ and $y < 6$.

Now we have,
$x < 6$........................(v)
$y < 6$........................(vi)
$z < 6$........................(vii)

If we take the value of x from (i) and substitute it in (v), we have, $12 - (y + z) < 6$ or $(y + z) > 6$

Similarly $(x + z) > 6$ and $(x + y) > 6$

Now we have the following information,
$x + y + z = 12$.....................(i)
$x < 6$...................................(ii)
$y < 6$...................................(iii)
$z < 6$...................................(iv)

$(y + z) > 6$............................(v)
$(x + z) > 6$ ....(vi)
$(x + y) > 6$...............................(vii)

Now, you can not take out anything more useful from these equations and inequalities. Here comes the real part of the problem.

(ii) says $x<6$. So, let us take some values for x:

Ist case,
$x = 1$
From (vi), $z > 5$ and from (iv) $z < 6$. It is clear that an integer value for z is not possible
Therefore this case is NOT POSSIBLE.
2nd case,
$x = 2$
From (vi), $z > 4$ and from (iv) $z < 6$. Therefore $z = \{5\}$
From (i), $y=12-(x+z)$ or $y=\{5\}$
3rd case,
$x = 3$
From (vi), $z > 3$ and from (iv) $z < 6$. Therefore $z = (4, 5)$
[i.e.z=4 or 5]
From (i), $y=12-(x+z)$ or $y=\{5,4\}$
4th case,
$x = 4$
From (vi), $z > 2$ and from (iv) $z < 6$. Therefore $z = \{3, 4, 5\}$
From (i), $y = \{5, 4, 3\}$
5th case,
$x = 5$
From (vi), $z > 1$ and from (iv) $z < 6$.Therefore $z =\{2,3,4,5\}$
From (i), $y = \{5,4,3,2\}$

From the above data, a table can be made:

x	y	z
2	5	5
3	5	4
3	4	5
4	5	3
4	4	4
4	3	5
5	5	2
5	4	3
5	3	4
5	2	5

x, y and z can be any sides. Therefore the unique values of x,y and z are: 2, 5, 5 and 3, 4, 5 and 4, 4 ,4

2. An isosceles triangle is one which has any two sides equal. An equilateral triangle is one which has all three sides equal and a scalene triangle is one which has no side equal. Matching this to the data, we have:
12 inches by 30 inches by 30 inches — isosceles
18 inches by 24 inches by 30 inches — scalene
24 inches by 24 inches by 24 inches — equilateral

Teaching Suggestions

When we first offered this problem the most common approach was guess and check. Many students noted in their explanations that they actually modeled the problem by using a folding ruler and some also used toothpicks to think about what could happen.

Most of the students who submitted a solution were able to answer the problem correctly. However, we found that many students were not verifying that their answers formed triangles, i.e., they did not show that they satisfied the “Triangle Inequality Property.” Those students who were not able to find the correct answers had two common errors. The first was that they did not understand that the ruler only bent at 6" increments, so that all dimensions had to be multiples of 6". The other common error was that they did not understand the Triangle Inequality Property. Therefore, 12" by 24" by 36" was a popular incorrect answer for the dimensions of the scalene triangle. Our typical suggestion to a student was to try modeling the problem using 12 toothpicks, where each section represents a 6" section of the ruler.

Note that the underlying concept for this problem is the Triangle Inequality Property. Some students might refer to it by name but others will acknowledge the thinking behind it without actually naming the property.

The questions in the Answer Check, above, might serve as good prompts to help students make progress, particularly the one link to Ask Dr. Math. Encourage students to use a strategy that works for them. You can see from the various methods that we have thought to use for this problem that there are several ways to approach it. And keep in mind that we may not have thought of them all!

Sample Student Solutions focus on Completeness

In the solutions below, I've provided the scores the students would have received in the **Completeness** category of our scoring rubric. My comments focus on what I feel is the area in which they need the most improvement.

Novice	Apprentice	Practitioner	Expert
Has written very little that explains how the answer was achieved.	Submitted explanation without work or work without explanation. Leaves out enough details that another student couldn't follow or learn from the explanation.	Explains all of the important steps taken to solve the problem, which might include explaining: • how they used the information given in the problem. • any relationships used. • the rationale behind each decision they made.	Adds in useful extensions and further explanation of some of the ideas involved. The additions are helpful, not just "I'll say more to get more credit."

Austin
age 12

Completeness
Novice

36 inches each

i tried it my self wih 2 rulers and mesherd it wit another

I notice Austin thought to use rulers to think about the problem but I'm not sure why he used only two.

I wonder if he had ten more rulers to think about the twelve sections if that might help. It's actually hard to tell from the little bit of explanation he provides, what exactly he was "trying" with the two rulers! I might start by just asking him to tell me more about that.

Erin
age 11

Completeness
Novice

The dementions to the three triangles are, 24in.,24in.,24in. The second triangle, 18 in, 30 in, and 24 in. The last triangle is, 30 in, 30 in, and 12 in.

I found the answer by creating a chart so I could visualy see it. Then, I found the the ways to fold the rulers.

Unlike Austin, Erin seems to have interpreted the problem correctly although she hasn't identified the types of triangles.

I would like to either see or hear more about her chart. I would ask her what labels she used and maybe how many columns or rows the chart had.

Kenneth
age 10

Completeness
Novice

The dimensions of the triangles are: 24x24x24, equilateral; 23x23x26, isosceles, 24x22x26, scalene.

I got my answer by adding up three different numbers that had the sum of 72.

I notice Kenneth worked with numbers that summed to 72 but, perhaps, didn't consider the restrictions that the folding ruler imposed on the problem.

I wonder how he might respond if I asked him about the 6 inch sections of the folding ruler. How do we bend the folding ruler so that a triangle's side measures 23 inches?

Justin
age 13

Completeness
Apprentice

the anser is for question one is 12,24,36 and 30,30,12 and 24,24,24.for question 2 the awnser is scalene-12,24,36.isosceles-30,30,12 for equolateral- 24,24,24.

i got my awnser by guess and check i first divide 72 by 3 (because of the three sides) which equals 24 and since all equal sides that became the equilateral.I then guess to equal side and one non equal that add up to 72 i got 30,30,and 12 and since it is two equal sides i got a isosceles.Then i guess mutilplies of 6 until i got 12,24,36 which equals no equal side so its a scalene

Justin has the right idea about how to use the numbers to generate three that multiples of 6 that add to 72 … but … he's left out one minor detail and that is if they will actually make a triangle.

I might ask Justin to try modeling the idea with 12 pencils (or toothpicks or sticks). He could use 2 pencils for the 12" side and 4 pencils for the 24" side and 6 pencils for the 36" side. Can he make a triangle?

Nicole
age 12

Completeness
Apprentice

1. The dimensions are:24x24x24,30x30x12,and 18x24x30. The triangle formed by to unfolded rulers is the Equilateral.The triangle that is formed by the stack of rulers and one unfolded one is Isosceles. The giant one is Scalene.

I found that 72 divided by 3 is 24. That means the equilateral is 24x24x24. Then I took the measurements of the ruler and found the Isosceles is 5x5x2. Then I multiplied it by 6 and got 30x30x12. Lastly, I took the measurements of the Scalene and they were 3x4.Then I found the hypotnuse and got 5. Then I multiplied it by 6. I got 18x24x30.

Nicole has a good start on her explanation but reflecting and revising will probably improve both her Completeness and her Clarity scores.

She appears to move back and forth between thinking of the sections of the folding ruler and considering the lengths in inches.

Tim
age 13

Completeness
Practitioner

The equilateral triangle has three sides of 24 inches each, the sides of the isosceles triangle are 30 inches, 30 inches, and 12 inches, and the scalene triangle has sides of 18 inches, 24 inches, and 30 inches.

I drew diagrams of what the ruler would look like when bent into triangles. I knew that equilateral triangles have all equal sides, isosceles triangles have 2 sides in common, and scalene triangles have all different sides. With this prior knowledge, I was able to draw three triangles with perimeters of 72 inches each.
Revision
I drew scale diagrams of the 12 sections of ruler where 1cm=6inches. Finding the equilateral triangle was easy-I just divided by 3 and came up with 4 sections for each side. For the isosceles triangle, I tried different combinations of sections and only 1 worked to form a triangle.(3+3+6 didn't work, 5+5+2 worked)
For the scalene triangle, 2+4+6 didn't work, 3+4+5 worked.
My reflection was that while working I observed that the closer the dimensions of the sides to each other, the more likely to be able to form a triangle.

I notice that Tim has labeled the second part of his explanation as "Revision" which indicates that he received feedback from a mentor and he used it!

His first submission had a score of "Apprentice" in Completeness and the mentor asked if he might "…show the steps of how you got your answer because this will help fellow students understand your thinking better." It's great that Tim took the time to add to his solution.

Elizabeth
age 12

Completeness
Practitioner

If each triangle formed by the folding ruler had the perimeter of 72 in. you could make a scalene, isosceles, equilatoral triangle. To make an equilatoral triangle you would have the dimensions of 24in. per side. For a scalene triangle you would have the dimensions of 23in, 24 in, and 25in. For a isoscles triangle you would have the dimensions of 26in, 23in, and 23, in. But all of them would have the same perimeter of 72in.

If you had a folding ruler, and made three triangles out of that ruler, each being either isoceles, scalene, or equilatoral, that all had the perimeter of 72in. what would the dimensions be. Well to find the answer to that, I took a few steps.

First off you must know the definition for each triangle. An equilatoral triangle had all equal sides. An isoceles triangle has 2 equal sides. Finally a scalene triangle has no equal sides.Knowing that we can preceed to the dimensions.

We know that a triangle has three sides and so to find out what the dimensions are for them are to be equal we must divide by the number of sides.
So:
72/3=24

So the equalatoral triangle has 24in. for each side of the triangle.

For the isoceles triangle two of the sides must be equal. So I first I knew that 24 couldn't be the answer because all sides would be equal. So I tried the next number down 23. I added 23 and 23 together and got 46. I then subtracted 46 from 72 and got 26. So the dimensions of the isoceles triangle is 23in, 23in, and 26in.

The scalene triangle was a bit easier because I knew that no sides had to be equal. And so using logics I figured that one or more had to be greater than 24, and one or two lower then 34.

First I tried 26, 25, and 24 but I found out that they totalled up to 75. Then I tried 24, 25, and 23 and they totalled up to 72.

Elizabeth, like Tim has included all of the important steps taken to solve the problem. I might suggest that she refer to the text of the problem to correctly spell "equilateral" and "isosceles" but that would be to improve her Clarity score and not her Completeness score.

Mac
age 10

Completeness
Expert

My solution for "Forming Triangles from a Folding Ruler" is that the equilateral triangle has lengths of 24 inches, the isosceles has lengths of 30 inches, 30 inches, 12 inches, and the scalene triangle has lengths of 24 inches, 18 inches, and 30 inches.

I needed to find out what the dimensions of the three different triangles were. First, I figured out the equilateral triangle because equilateral triangles have three equal sides and angles, so I only had to divide 72 inches (6 feet) by 3.

72÷3=24inches so the equilateral triangle has lengths of 24 inches

Next, I found the scalene triangle by adding and subtracting 6 from 24 because the folding ruler is made up of 12 sections of 6 inches each. A scalene triangle has no equal sides or angles.

24+6=30inches 24-6=18inches so the scalene triangle has lengths of 30 inches, 24 inches, and 18 inches

Finally, I found the isosceles by getting 3 yardsticks and making isosceles triangles until the triangle was isosceles and a multiple of 6. So, the isosceles triangle has lengths of 30 inches, 30 inches, and 12 inches. This fits an isosceles triangle because an isosceles triangle has two equal sides and angles.

I used a rule called the triangle inequality property. It means that if you add any of the two dimensions, you'll have a bigger number than the third dimension. I used this rule for the scalene and isosceles triangles.

For example 30+12=42 (bigger than 30)
30+30=60 (bigger than 12)
30+24=54 (bigger than 18)
30+18=48 (bigger than 24)

Before (when I did not know the rule) I had the isosceles triangle at 30 in., 21in., 21in for the dimensions. This number did work for the rule, but 21 is not divisible by six. So, I tried different combinations for the dimensions and they still did not work. So, I went on the internet and found the rule. Then, it was easy to find the dimensions for the isosceles triangle.

Mac has written a very complete explanation explaining his thinking as he works through the solution. Great job!

I might suggest a few formatting suggestions because with a few more spaces here and there his Clarity score could be improved but as is it's still quite readable!

Scoring Rubric

A **problem-specific rubric** can be found linked from the problem to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work.

We hope these packets are useful in helping you make the most of Pre-Algebra Problems of the Week. Please let me know if you have ideas for making them more useful.

https://www.nctm.org/contact-us/