NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Problem of the Week Teacher Packet

So Many Salmon

On February 17, 2004, an article in *The Sacramento Bee* reported that hatcheries from Alaska to California tag fingerling salmon with nose tags so that researchers can learn more about where the fish go during the year. About 45 million salmon are tagged each year. That's a lot of fish!

Armida Soria, a worker at the hatchery company owned by Jerry and Diane Big Eagle, can tag about one fish every four seconds during her 8-hour shift. The article that focuses on the Big Eagle hatchery ends with this phrase: "Another day, another 70,000 salmon."

Question: If the Big Eagle runs two shifts a day with six workers per shift, is the average worker faster or slower than Armida?

Extra: When a person is first learning to do this tagging, it takes about 10 seconds to tag each salmon. In one day (an 8-hour shift), what percentage of Armida's salmon will the new person tag?

Answer Check

After students submit their solution, they can choose to "check" their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually get the answer) we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

> *The average worker is slower than Armida. (In fact the average worker's time is about 1 second slower than Armida's average time per fish.)*

If your answer does not match our answer,

- instead of finding the average time in seconds, did you find out how many fish the average worker tagged in a minute? (you can compare that way too)
- did you include 12 workers in your calculations since there are two shifts?
- have you checked your calculations?
- have you reread the problem to make sure you answered the questions asked?

If any of those ideas help you, you might revise your answer, and then leave a comment that tells us what you did. If you're still stuck, leave a comment that tells us where you think you need help.

If your answer does match ours,

- does this problem remind you of any real experiences you've had?
- are there any hints that you would give another student?
- did you make any mistakes along the way? If so, how did you find them?
- are you confident that you could solve another problem like this successfully?
- did you reread your explanation to see if you could make it clearer or more complete?
- did you try the Extra question?

Revise your work if you have any ideas to add. Otherwise leave us a comment that tells us how you think you did—you might answer one or more of the questions above.

Our Solutions

Method 1: Notice and Wonder

Our group used the Activity Series worksheet that our teacher gave us and tried to notice and wonder everything that we possibly could. Here was our list:

We noticed:

- 45 million salmon are tagged each year.
- Armida works 8-hour shifts.
- Armida tags one fish every four seconds.
- The workers in the hatchery tag 70,000 salmon a day.
- There are two shifts that work each day.
- There are six workers per shift.

We wondered:

- What do they want us to find?
- Is there a good way to organize the information given in the problem?
- Are there any pictures that we can draw to help us think about the story?
- Is there a way we could act it out?
- Could we make the problem simpler? How?
- What are the relationships between the numbers?
- Should we change all the numbers to seconds or to days?

We remembered:

- There are 60 seconds in 1 minute.
- There are 60 minutes in 1 hour.
- Using their information, there are 8 hours in 1 day.

We started by thinking about Armida. If she tags one fish every four seconds, then using what we know about how many seconds are in a minute that means she tags 15 fish each minute (60 ÷ 4 = 15). And that also means that in 1 hour Armida tags 900 fish (60 x 15) and in 8 hours that would be 7200 fish. We thought it a little strange to think of Armida working non-stop for 8 hours straight but to make the comparison we figured it would be okay to think about it this way.

The workers in the hatchery together tagged 70,000 and if there were two shifts of six workers each that makes 12 workers total. To find how many fish they each tagged, we divided 70,000 by 12. We got 5833 1/3 fish. When we compare that amount to Armida's amount, she tags more fish. We conclude the average worker is slower than Armida.

Method 2: Make a Table

After we read the problem, we thought it might help to keep track of everything in a table. Here is what we put in our table:

Worker	*Tagged per 4 seconds*	*Tagged per minute (60 ÷ 4)*	*Tagged per hour (times 60 because there are 60 minutes in 1 hour)*	*Tagged per day (times 8 because there are 8 hours in 1 workday)*
Armida	1 fish	15 fish	15 x 60 = 900 fish	900 x 8 = 7200 fish
12 workers	?	?	?	70,000 fish
1 worker	?	?	?	70,000 ÷ 12 = 5833.33… fish

Because 7200 is a bigger number than 5833, we decided that the average worker at the hatchery is slower than Armida.

Extra: If a new worker is learning, they will have 28,800 seconds (60 x 60 x 8) in their 8 hour shift. If they tag one salmon every 10 seconds, they will tag 2880 salmon. They tag 2880 salmon and Armida tags 7200 salmon in the same time. 2880 out of 7200 is (2880 ÷ 7200) 0.4 or 40% of what Armida could tag.

Method 3: Work Backwards

After reading the problem I know that:

- 45 million salmon are tagged each year.
- Armida works 8-hour shifts.
- Armida tags one fish every four seconds.
- The workers in the hatchery tag 70,000 salmon a day.
- There are two shifts that work each day.
- There are six workers per shift.

I decide to calculate how many salmon Armida tags in one day so that I can compare that to the other workers.

First I multiply to find how many fish she tags in one minute since 60 seconds = 1 minute:

4 seconds x 15 = 60 seconds so (1 fish each 4 seconds) x (15) = 15 fish each 60 seconds

Next I find out how many fish she tagged in an hour since 60 minutes = 1 hour:

15 fish x 60 = 900 fish each hour

At first I wasn't sure if I should multiply or divide by 60 but it makes sense that she tags more fish in an hour than she does in a minute so I multiplied to get a bigger number.

If she works an 8 hour day, then

900 fish x 8 = 7200 fish each workday

If there are 12 workers tagging as fast as she does, then they would tag

7200 fish x 12 = 86,400 fish

That's more than 70,000 and so that means that the average workers are slower than Armida.

Method 4: Proportional Reasoning

The hatchery tags 70,000 fish a day. It does this with 2 shifts, 6 workers per shift. This means that there are 12 workers that tag 70,000 fish in one day.

2 shifts/day • 6 workers/shift = 12 workers/day

This means that each worker tags 5,833 1/3 fish per day.

$$\frac{70,000 \text{ fish/day}}{12 \text{ workers/day}} = 5,833\frac{1}{3} \text{ fish/worker}$$

Each worker has an 8-hour shift, so I can do a few more calculations to figure out the average time it takes to tag one fish:

$$\frac{5,833\frac{1}{3} \text{ fish / worker}}{8 \text{ hours / worker}} = 729\frac{1}{6} \text{ fish / hour}$$

$$\frac{729\frac{1}{6} \text{ fish / hour}}{60 \text{ minutes / hour}} = 12\frac{11}{72} \text{ fish / minute}$$

$$\frac{1 \text{ minute}}{12\frac{11}{72} \text{ fish / minute}} = \frac{60 \text{ seconds}}{12\frac{11}{72} \text{ fish / minute}}$$

$$= 4.93... \text{ seconds / fish}$$

It takes an average of a little less than 5 seconds to tag each fish. The average worker's tagging is slower than Armida's tagging.

Check - if my calculations are correct, I should be able to multiply 5 seconds by the other factors to come up with a number that is less than 70,000 fish:

1 fish/5 seconds • 3,600 seconds/hour = 720 fish/hour

720 fish/hour • 8 hours/worker = 5,760 fish/worker

5,760 fish/worker • 12 workers/day = 69,120 fish/day

My work checks!

Extra: Armida takes about 4 seconds to tag each fish, while a new person starts at about 10 seconds per fish. I am to find the percentage of fish the new person tags using Armida as the base of 100%. One way to do this is to look at how many fish they'd each tag in the same amount of time. I'm going to pick 20 seconds, because 20 is a multiple of both 4 and 10. In 20 seconds, Armida will tag 5 fish.

20 seconds • 1 fish/4 seconds = 5 fish

In 20 seconds, a new person will tag 2 fish.

20 seconds ÷ 1 fish/10 seconds = 2 fish

Now I can look at the ratio,

Armida : New person

5 fish : 2 fish

= 5(20) : 2(20)

= 100 : 40

The new person will tag approximately 40% of the number of fish that Armida tags.

Standards

If your state has adopted the Common Core State Standards, you might find the following alignments helpful.

Grade 3: Operations & Algebraic Thinking

Represent and solve problems involving multiplication and division.

Grade 3: Measurement & Data

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

Grade 4: Operations & Algebraic Thinking

Use the four operations with whole numbers to solve problems.

Grade 4: Measurement & Data

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

Grade 5: Number & Operations in Base Ten

Perform operations with multi-digit whole numbers and with decimals to hundredths.

Grade 5: Measurement & Data

5.MD.1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.

Teaching Suggestions

When this problem was first offered, the most common general approach we saw was to calculate the average number of fish tagged in a day by one worker as well as the number of fish tagged in a day by Armida, and then compare these numbers. Because there is a lot of numerical information given in the problem and also time conversion ideas to take into consideration, using the Scenario [pdf] as a starting point could help engage students in conversations.

There are slightly different ways of arriving at the average number of fish tagged in a shift by one worker. One method is to first calculate the number of people working, then divide by the result. Another method is to first divide the number of fish per day into shifts, then continue using the information about how many people worked each shift.

We noticed with the submissions in 2004 that the most common errors for this problem included:

- miscalculating the number of seconds per hour
- leaving out a step in the calculations
- multiplying instead of dividing (or vice versa) in a conversion
- using incorrect unit conversions

Sample Student Solutions – Focus on *Completeness*

In the solutions below, I've provided scores the students would have received in the **Completeness** category of our scoring rubric. My comments focus on areas in which they seem to need the most improvement.

Novice	Apprentice	Practitioner	Expert
Has written very little that explains how the answer was achieved.	Might show that their solution works without saying anything about how they figured it out.	Tells all of the important steps taken to solve the problem, which should include: • some indication of the need to convert the different units of time given in the problem. • how they compared the number of fish tagged which might include: • fish/day (whole factory) to sec/fish (per worker) • sec/fish for Armida to fish/day for "12 Armidas" • explaining why their answer is correct.	Adds in useful extensions and further explanation of some of the ideas involved. The additions are helpful, not just "I'll say more to get more credit."

Aaron, age 10, Novice

the answer is the average worker is slower than Armita.

i read it, then wrote it out on paper then i answed on the computer

On one level Aaron has told us what he did to solve the problem but mathematically he hasn't communicated any of his thinking.

To prompt him to say more I would ask him to list five things he noticed when he read. I would

ask him that instead of asking him about what he wrote because it's less threatening and should get him back into thinking about the problem.

Spencer, age 12, Novice

The average worker is slower than Amida.

I looked at the extra problem, and it gave me the answer.

Comment: I had the correct answer, and why do you have to make it be so sophisticated for it to be counted right

The comment Spencer left after viewing the answer check provides an opportunity to discuss why communication is required for the PoWs. As an online mentor I often explain that I have to completely rely on what students write to understand what they are thinking. I can't talk to them in person!

If I were Spencer's teacher and could talk to him in person, I would point out that the PoW rubric has two parts, one that focuses on "problem solving" and one that focuses on "communication." It's not a matter of a sophistication but rather covering both areas of problem solving.

Robert, age 13, Apprentice

No the average worker is not faster than Armida

First I mutiplied (60 times 60)times 8=28800 divided by 4=7200 fish per 8 hour shift. Then I took 70000 and divided by 12 which was 5833.333333 and as you can see I didn't have to go any further.

Because I'm familiar with the problem I recognize the numbers Robert is using and I can see that he has a great start.

I might remind him that he should think about a classmate reading his solution. Has he included enough of his thinking so that they can really understand the problem? Why did he multiply 60 times 60, for example?

Tyler, age 9, Apprentice

Armida tagged 1,380 salmon.The other workers tagged 70,000 salmon, she was slower then the other workers

I solved it by dividing 60 and 4 it = 15 salmon were tagged.There

I notice that Tyler left out either the idea of seconds or the idea of minutes. It's a little difficult to

worked tagging in 8 hour shifts. I mutilplide 15 salmon x 8 hour shift =115 salmon. Then iI multiplied 115 x 6 workers= 690 salmo that were tagged

know because he's not always included units with his numbers.

I might point out that he used "hours" to label his 8 and wonder what label he would use for the 60 or the 15 or the 4.

Connell, age 9, Apprentice

Armminda works more then the average worker. New people do 40% of Arminda's work

The first thing i did was divide 60 by 4 (15) because it would tell me how many fish arminda would tag in a minute.
I multiplied that by 60 to get how many fish she tags in an hour(900)
I multiplied that by 8 to get how many fish she tags in her shift. (7,200)
If Arminda works more than one 12th of the fish caught in a day (70,000)than she works more than the average worker because 12 people
work each day including Arminda.
7,200 is more than one twelth so Armida works more than the average worker.

For the extra,
4 tenths is the same as 40 hundreths which is th same as 40%

Connell has a great start in his explanation. As he notes his calculations he includes why he did each step and what he was thinking.

I notice at the end he compares Armida's number of tagged fish to 1/12 of the other workers' tagged fish but he doesn't give us that number so that we also can see the comparison. With that additional information the main part of Connell's explanation would be complete. He has a start on his Extra but I would like to see more of his thinking.

Megan, age 9, Practitioner

The average worker at Big Eagle is slower than Armida.

The average worker at Big Eagle is slower than Armida. In order to solve this problem I first figured out how many workers there are at the Big Eagle during one day. I solved this by multiplying 6 workers by 2 shifts a day. The answer is 12. Then I needed to determine how many fish the average worker tagged in a a day. I solved this problem by dividing 70,000 fish divided by 12 workers. The answer is each worker tags an average of 5,833 fish per day. Then I had to figure out the average amount of fish Armida fish tagged per day. I knew that she could tag one fish every four secons and that she worked 8 hours a day. So I multiplied 8 hours by by 60 seconds to det,ermine how many seconds were in Armida's 8 hour shift. The answer is 28,800 seconds. Then I divided 28,8000 seconds by 4 seconds. The answer is, Armida can tag 7,200 fish in an day. Lastly I compared Armoda's average of 7,200 to the average worker's of 5,833. Another way to calculate Armida's daily average is to divide 60 seconds by 4 to determine the average number of fish she could tag in a minute. The answer is 15. Then I multiplied 15 fish times 60 minutes to determine the number of fish she tagged in an hour. The answer is 900. Then I multiplied 900 times 8 hours to determine the number of fish per day. The answer is 7,200 fish per day.

Megan has done a complete job of explaining what she did and why. I would suggest that she work on her Clarity next by adding in a few paragraph breaks here and there and also, perhaps, try the Extra.

Matt, age 11, Expert

The average worker is slower than Armida. In one day (an 8-hour shift), the new person will 40% of Armida's salmon.

I took these steps to get my answer:

1. I figured out how many seconds 8 hours is by multiplying it by 60 twice, or 60^2. (8 hours * 60 * 60 = 28,800 seconds)

2. I figured out how many salmon Armida could tag by dividing my answer by four because Armida can only tag one salmon every four seconds. (28,800 seconds / 4 = 7,200 salmon)

3. I divided 70,000 salmon by 2 shifts and then by 6 workers to find out how many salmon the average worker tags every day. (70,000 salmon / 2 shifts / 6 workers = 5,833 1/3)

4. I compared 5,833 1/3 (my step 3 answer) with 7,200 (my step 2 answer) to get my answer. (7,200 for Armida > 5,833 1/3 for average worker)

Extra: I divided 28,800 by 10 to find out how many salmon a new person could tag in one 8-hour shift. (28,800 seconds / 10 = 2,880 salmon). I then divided this by 7,200 and multiplied this answer by 100 to get my answer. (2,880 / 7,200 * 100 = 40%)

One advantage to being able to look in the "back office" for students' solutions is that I can see that Matt didn't have this complete and clear solution on his first try.

He actually had four exchanges with his mentor before getting to this final draft. He took to heart the idea that problem solving and communication is not something to finish and be "over and done" but instead it is a process!

Scoring Rubric

A **problem-specific rubric** can be found linked from the problem to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work.

We hope these packets are useful in helping you make the most of Math Fundamentals Problems of the Week. Please let me know if you have ideas for making them more useful.

https://www.nctm.org/contact-us/