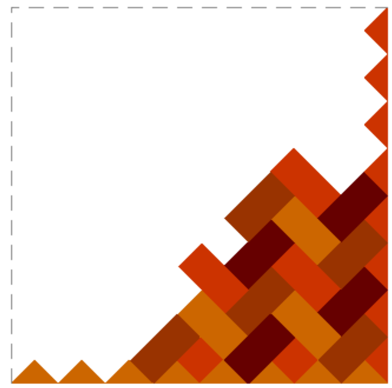
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Problem of the Week Teacher Packet

Women's Walkway

On the brick "Women's Walkway" from the intersection of 33rd and Chestnut to the intersection of 34th and Walnut in Philadelphia, I became fascinated with the pattern of the bricks.

Here is an area that I'm filling with the pattern:



I thought it was economical how some bricks were cut in half and some of the halves were also cut in half to neatly fit to the edges.

If the top face of the uncut bricks are 2 units by 1 unit,

1. how many bricks would it take to cover the full area of the pattern shown above?
2. what is the area?

Answer Check

After students submit their solution, they can choose to "check" their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually get the answer) we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

64 uncut bricks would be needed.

If your answer does not match our answer,

- did you notice that "cover the full area of the pattern" means that you would continue placing bricks until they reach the dotted lines?
- you might want to try drawing or using a manipulative to think about the problem?
- did you notice that four triangle-shaped pieces of brick make up one uncut brick?
- did you notice that two square-shaped pieces of brick make up one uncut brick?
- did you look for ways to make a simpler problem?
- have you checked your arithmetic?

If any of those ideas help you, you might revise your answer, and then leave a comment that tells us what you did. If you're still stuck, leave a comment that tells us where you think you need help.

If your answer does match ours,

- have you also answered the **second** question? (Note we only gave the first answer here in the Answer Check.)
- are you confident that you could solve another problem like this successfully?
- is your explanation clear and complete?
- did you make any mistakes along the way? If so, how did you find them?
- what hints would you give another student trying to solve this problem?

Revise your work if you have any ideas to add. Otherwise leave us a comment that tells us how you think you did—you might answer one or more of the questions above.

NOTE: We decided not to include these questions in the Answer Check that we have made available to students, but if you are using some of the **Solve a Simpler Problem** activities included in the **Activity Series** document, you might consider using some of these questions with your students.

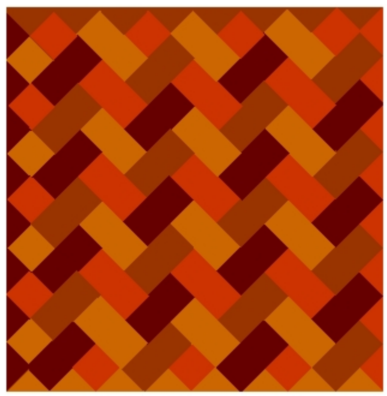
If you used "Solve a Simpler Problem," did you explain

- how you identified what was hard?
- how you decided what to consider in making it simpler?
- how you made a connection from your simpler problem back to the full problem?

Our Solutions

Method 1: Using a Model or Drawing

To think about how many bricks would be needed, I'm going to think about how many whole, half, and quarter brick would be used. To do this I first completed the pattern so I could see how many bricks would fill certain types of sections (if this is the case, it would be nice if the submitter added an upload of the picture they created).



To help talk about the brick, I'm going to "define" each color. Referring to the picture in the puzzle, I started with the lower right corner. It's made up of two quarter-bricks, and they look tan and orange to me. The brick just above them I'll call sienna, with the fourth and final color above that being brown.

After I finished the picture, I decided to count whole bricks first. On the right side of the pattern, the whole bricks stack up sienna, brown, sienna, brown... which continues for seven (7) bricks - these bricks are oriented "upward" in the sense that they sort of travel from lower-left to upper-right.

Next, there's a stack to the left of this stack that goes tan, orange, tan, orange..., but this stack travels in a different direction - "downward" in the sense that they sort of travel from upper-left to lower-right. There are seven (7) bricks in each of these stacks as well.

In my completed pattern, there are four (4) sienna-brown upward stacks and three (3) tan-orange downward stacks, so that's seven (7) stacks of seven (7) bricks each. I can multiply to figure out the total number of whole bricks used:

7 stacks * 7 whole bricks/stack = 49 whole bricks

Now I'll look at half-bricks. At the bottom of my picture, there are three (3) orange half-bricks (the two that were in the puzzle picture as well as one more I colored in my picture).

At the top of my picture, there are four (4) orange half-bricks.

There weren't any on the right side of the pattern, but there are seven (7) on the left side, alternating tan and orange. Now I just add them all up, then divide by 2 to make whole bricks (pairing half-bricks to make a whole):

3 + 4 + 7 = 14 half-bricks
14 half-bricks = 7 whole bricks

Finally, I need to count quarter-bricks. There are eight (8) quarter-bricks on each side of the pattern sample.

There are four (4) sides of the pattern, so I can multiply to find the total number of quarter bricks, however, I also group them by fours to count whole bricks - so this is eight (8) more whole bricks:

8 quarter-bricks/side * 4 sides = 32 quarter-bricks
32 quarter-bricks = 8 whole bricks

Now I add them all up for my final sum:

whole bricks + pairs of half-bricks + groups of quarter-bricks
49 + 7 + 8
64 bricks

It takes **64 bricks** to make the pattern sample.

NOTE: Students who previously submitted solutions when this problem was used in 2004 noted that they counted 7 (square) half-bricks in their drawing or model and not the 14 that we counted. Because they did not include a scanned drawing or photograph, we weren't sure how they "saw" the pieces of brick. One idea we had was that they thought of ¾ pieces of brick, instead of just ½ and ¼ pieces. If they arrive at the final total of 64 bricks and their explanation seems reasonable, do not count off for not having the exact same description as what we have provided here.

To find the total area, I multiply the area of one brick by the number of bricks needed:

1 brick = 2 units by 1 unit
= 2 units^2
64 bricks * 2 units^2/brick = 128 units^2

The area of the pattern sample is **128 units^2** which may also be represented as 128 units^2, 128 un^2, or 128 sq. un.

Method 2: Using Solve a Simpler Problem

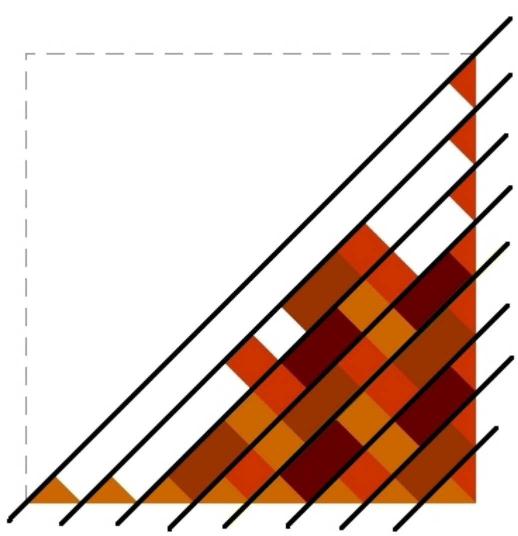
Working from a corner

The lower right corner of the pattern in the picture, is a 1/4 (of the 1 by 2) brick fit together with another 1/4 brick which together makes a 1/2 brick.

Looking at the next diagonal a full brick was added. In that diagonal there is one full brick with a 1/4 brick on one end and another 1/4 brick on the other end.

I continue looking at the diagonals in the pattern and there are 8 in half of the pattern in the square and when I think about how many bricks are in each diagonal, I get

1/2 + 1 1/2 + 2 1/2 + 3 1/2 + 4 1/2 + 5 1/2 + 6 1/2 + 7 1/2.

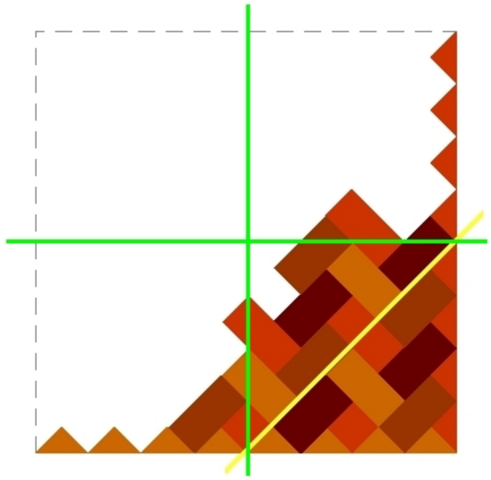


This is a total of 32 bricks in half of the square. When I double that number to find the number of bricks in the entire square, that number is 64.

To answer the second question, I know that the dimensions of each brick are 1 unit by 2 units. Each brick has an area of 2 square units and 2 square units x 64 = 128 square units.

Looking at an eighth of the pattern

As I look at the sample pattern, I wonder what the largest fraction is that I might work with that has all of the bricks filled in. If I consider dividing the sample pattern into four equal parts (using the small triangular pieces as a guide), then cut one of those "fourths" in half diagonally, I have an eighth of the pattern.



I count the bricks that I can see and I have:

eight 1/4 pieces of brick (the triangle-shaped bricks)

two 1/2 pieces of brick (the square-shaped bricks)

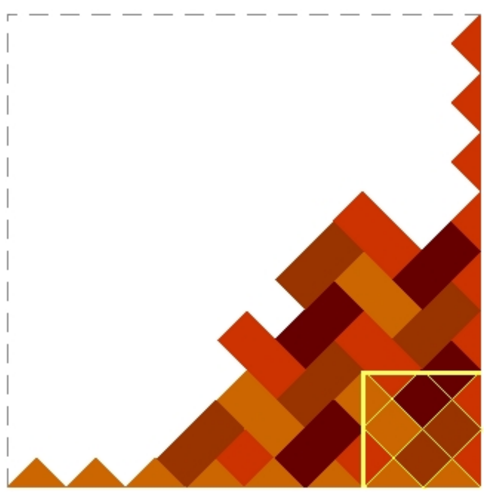
five whole pieces of brick (the rectangle-shaped bricks)

Adding those together, I have the equivalent of 8 bricks. Since I'm considering just an eighth of the complete sample pattern, the full pattern uses 64 bricks (some cut in half and some cut into fourths and some used whole).

If a brick is 1 unit by 2 units, then I know that the area of the top is 2 square units for each whole brick. If I need 64 whole bricks to fill the sample pattern, then 64 x 2 square units is 128 square units.

Looking at a sixteenth of the pattern

First, I divided the pattern sample with horizontal lines at the peaks and indents of the triangles bordering the right side. In the space between the bottom of my first line I counted the bricks. Each triangle (pointing upward and downward) counted as 1/4 and each square counted as a 1/2.



After adding this I got a total of 4 bricks, so I knew that there were 4 bricks in each of the 16 sections.

Then, I multiplied 16 by 4 and got 64 bricks to cover the full area of the pattern sample.

Next, I knew that the shaded triangles on the bottom were 45-45-90 triangles because there was one right angle and two other equal sides. The bottom sides each equaled the square root of 2, and when I added 8 of these I got 8 square root of 2. Then, I squared this and got 128 square units for the area.

Method 3: Using the Pythagorean theorem

It seems to me that is would be easier to answer the second question first, then divide by the area of 1 brick, so I'm starting with the area of the entire walkway.

I know the basic brick unit is 2 units by 1 unit. If I cut one of these in half, I have a square that is 1 unit by 1 unit. Cutting that in half diagonally to form the final type of piece in the picture, I create a right triangle with legs each 1 unit. The hypotenuse of the triangle is against the edges of the pattern, so I can find it's length and then multiply to find the length of the entire area. I'll use the Pythagorean theorem, which says the first leg squared plus the second leg squared is equal to the square of the hypotenuse of the right triangle:

```
leg^2 + leg^2 = hypotenuse^2       (I'm going to call this h^2)
    1^2 + 1^2 = h^2
        1 + 1 = h^2
            2 = h^2
        sqrt2 = h
```

The edges of the pattern sample are each 8 diagonals or 8 hypotenuses long, so I can multiply by 8 to find the length:

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 8 * sqrt2 = 8 sqrt2
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To find the area of the pattern sample, I multiply length times width (in this case the same since the pattern sample is square):

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length * width = 8 sqrt2 * 8 sqrt2
               = (8 * 8)(sqrt2 * sqrt2)
               = 64 * 2
               = 128 units^2
```

So the area of the pattern sample is **128 units^2** which may also be represented as 128 units^2, 128 un^2, or 128 sq. un.

Now back to the first question - how many bricks would be needed to make the complete pattern sample? Each brick is 2 units by 1 unit, so the area of 1 brick is 2 units^2.

2 units * 1 unit = 2 units^2

The entire area is 128 units^2, so I can divide by area per brick to find number of bricks needed:

128 un^2 / (2 un^2/brick) = 64 bricks

64 bricks are needed to make the square pattern sample.

Standards

If your state has adopted the Common Core State Standards, you might find the following alignments helpful.

Grade 6: Geometry

Solve real-world and mathematical problems involving area, surface area, and volume.

Grade 7: Geometry

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Grade 8: Geometry

Understand congruence and similarity using physical models, transparencies, or geometry software.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.

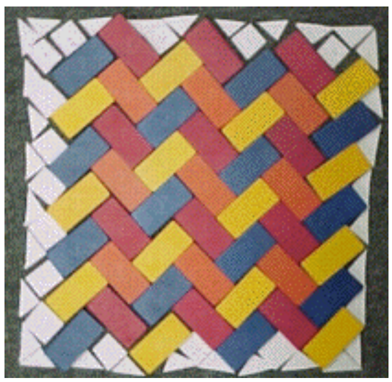
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Teaching Suggestions

Many students solved this problem successfully when we first used it in 2004, either trying to complete the pattern by visualizing or drawing or using a manipulative. Not all students understood that the pattern was important and since we didn't really make that a requirement, as long as the sample are has been filled, we couldn't actually count off if the students didn't realize the tessellation.

When this problem was first used in 2004, a group from Hope College volunteered their mentoring services as they worked with their professor in their class, *Problem Solving for Elementary Teachers.* As they prepared to mentor the student submissions, they worked the problem themselves and came up with these two diagrams. The one of the left, they used cut-out manipulatives and the one on the right is a drawing using dot paper.

We like these photos because they are accurate but not exactly precise. They let you talk more about the math but are just a model (left) and a drawing (right) to help students visualize the sample section of the walkway.



Also, they suggested some questions to use during a classroom discussion:

- How many bricks are there of each color? How many bricks in all?
- What about map coloring? Do bricks of the same color touch each other? How many colors are needed for this "map?"
- How does the pattern continue to fill the region shown?
- If you double the dimensions, how does it change the number of bricks needed?
- What would proportional size changes do to the design and number of bricks?
- What kind of symmetry is present in this pattern? Do the other corners that are not shown reflect or rotate to match this one?

What is the area of the whole region?

The questions in the Answer Check, above, might serve as good prompts to help students make progress, particularly the one link to Ask Dr. Math. Encourage students to use a strategy that works for them. You can see from the various methods that we have thought to use for this problem that there are several ways to approach it. And keep in mind that we may not have thought of them all!

Sample Student Solutions – Focus on *Strategy*

In the solutions below, I've provided scores the students would have received in the **Strategy** category of our scoring rubric. My comments focus on areas in which they seem to need the most improvement.

Novice	Apprentice	Practitioner	Expert
Has no ideas that will lead them toward a successful solution or shows no evidence of strategy.	Uses a strategy that uses luck instead of skill. Doesn't provide enough detail to determine whether it was luck or skill.	Uses a strategy that relies on skill, not luck, which might include: • using a model or drawing • logical reasoning • simpler problem	Uses two different ways to generate possible solutions.

Linda, age 12, Novice

It would take 66 bricks to fill the pattern sample area. The area of the pattern is 64.

The way I got the answer to this problem was like this:

-Since the pattern was in the shape of a triangle, I doubled the amount of peices used to make one half. Then that is how I got 66 for how many peices would be needed to make the completed pattern.

-For the area, there were 8 triangles on each side. I figured there would be 8 triangles on the other two sides since it was in the shape of a square. Then I multiplied 8 by 8 and got 64 for the area

Linda counted the bricks that are visible but hasn't indicated that she distinguished between uncut and cut bricks. I wonder if she might see the difference if she used a manipulative?

Linda has a method for finding area but I wonder how she knows that the base of the triangle she is using as her unit is one?

Elliot, age 11, Apprentice

49 whole bricks, 6 half bricks and 32 quarter bricks, or 60 whole bricks

Elliot drew bricks on the diagram and figured out that there would be 7 vertical columns of 7 whole bricks, 3 half bricks only at the top and bottom=6 and 32 quarter bricks, 8 bordering each of 4 sides = 32. If the answer is in whole bricks, it'd be 60 whole bricks

Elliot knows to count the bricks and he has an idea of how he can make a system to count them, namely his vertical column idea.

I might encourage Elliot to think more about his vertical column idea. Can he make a simpler problem? How many vertical columns make up the complete pattern? How many bricks are in each vertical column?

Brenda, age 12, Practitioner

1. 32 bricks are needed to cover the pattern sample.
2. The area of the sample is 64 square units.

To solve this problem I did a few things. First I found the lengths of the two legs of the triangle (which are both 8 times the sqaure route of 2) and mutiplied This came out to 128. Then I devided by two because I was trying to find the area of a triangle and not a square, and this came out to be 64 which is the area. To find how many bricks were needed I devided by two again since the bricks cover two square units. 32 bricks are needed to cover the pattern sample.

Brenda correctly states that a brick (I'm assuming she means an uncut brick) covers two square units. I wonder if she has noticed the cut bricks?

I also wonder why she changed to finding the area of the "triangle" instead of the full square pattern sample. Perhaps if she had 32 bricks as a manipulative she could check to see if her answer is reasonable.

Patrick, age 13, Practitioner

It would take 21 bricks to fill the walk way and the area is 21 bricks.

First I thought that each triangle is a fourth of a brick and each square is a half of a brick. There were to halves so that is one brick. There were 16 fourths so that is 4 bricks and then plus the other 16 that is 21 bricks which is the answer to both questions.

Patrick has a fine strategy for the interpretation that he has of the problem. It seems that he thinks that the task is to count how many bricks are visible in the graphic. It's not clear how he has decided on the area.

I would encourage Patrick to read the text of the problem again and I might ask him if he noticed the phrase "cover the full area of the pattern."

Minerva, age 12, Practitioner

64 bricks and 128 units

To find how many bricks it would take to cover the full area of the Pattern sample, I wanted to first find the total area of the pattern and then divide it by the area of 1 brick. The area of 1 brick is 1x2 =2 square units. To completly solve this problem I need to find the area of the pattern, that is solve problem 2.

To find the area of the pattern I divided each side of the pattern evenly by 4. I got a small pattern square whose side 1/4 the side of the whole sample pattern. The area of this small is then 1/4 x1/4 (=1/16) of the area of the whole sample pattern. I then calculated the area of the small square, by counting the bricks, 1/4 bricks in it and summing their area. The area of the small square was 8 square units. I then calculated the area of the whole sample pattern by multiplying times 16 (8x16=128). The area was 128 square units.

Then I went back to Problem 1. Since the area of the whole pattern is 128 square units, the number of bricks required to cover it was 128/2=64.

Minerva made a simpler problem by looking at a fraction of the pattern and counting the visible bricks. Once she knew that, she used a scale factor to find the complete area and then divided by the known area of bricks to find the number of uncut bricks needed.

I would ask Minerva questions to prompt her to explain how she counted the bricks. I wonder how she counted the uncut vs. cut bricks?

Cait, age 13, Practitioner

1) 64 Bricks
2) 128

To find out how many bricks covered the full area, i made a diagonal line through half the circle, added a few bricks following the pattern , and then counted all the bricks. for that i got 32 bricks, and then i had to double it, because it was only half of the circle. After doubling it, i got 64, which was how many bricks were in the circle.

To find the area, i knew that each unit was 2, and so the area of each unit was 2. I had to multiply 2 by 64, because length times width is how you find the area. 64 times 2 is 128. 128 is the area of bricks in the Women's walkway.

Cait also used a simpler problem method by looking at half the pattern. I wonder why he used the word "circle"?

I also wonder who he counted the bricks taking into account the different shapes.

Anne, age 14, Practitioner

Sixty four bricks are needed to complete the pattern. The area would be 128 square units.

The first corner was 1/2 brick. Each longer diagonal added a full brick. There are 8 diagonals in half of the square making a pattern of 1/2 + 1 1/2 + 2 1/2 +...+7 1/2. This is a total of 32 bricks and double for the entire square is 64. Each brick has an area of 2 square units. so 2 x 64 = 128 square units.

Although Anne has not spent too much time including details that would make it easier for the reader to visualize her strategy, the strategy itself uses an interesting method to use a pattern and a simpler problem.

Laura, age 9, Expert

1. It would take 64 full bricks to cover the full area of the pattern sample.

2. The area of the full pattern sample is 128 square units.

First, I decided to solve the problem by drawing. I drew 2 linesparallel to the edges of the pattern to form the area for the whole pattern square. Then I drew a diagonal line from the top right corner to the bottom left corner which split the square in half and made two equal triangles. Then I decided to finish filling in the bricks in the triangle that was already mostly filled in with the pattern.

When I finished filling in the pattern, I then counted up the number of whole bricks, half bricks and quarter bricks. There were 25 whole bricks, 6 half bricks, and 16 quarter bricks.

To find the number of whole bricks, I converted the number of half bricks and quarter bricks to whole bricks.

Since there a 2 half bricks in each whole brick, I divided 6 by 2 to get the number of whole bricks.
6 / 2 = 3 whole bricks.

Since there are 4 quarter bricks in each whole brick, I divided 16 by 4 to get the number of whole bricks.
16 / 4 = 4 whole bricks.

Then I added 25 + 3 + 4 = 32 whole bricks for half of the pattern sample. To get the whole, I multiplied 32 by 2 to get the number of bricks to cover the full area of the pattern sample.
32 * 2 = 64 bricks.

2. Since each brick is 2 units by 1 unit, its area covers 2 * 1 = 2 square units. Since there are 64 bricks, I would multiply them together to get 64 * 2 = 128 square units.

Laura used two strategies to solve the problem. She used a drawing combined with an idea of using a simpler problem. She also used the Pythagorean theorem even though she doesn't formally identify it.

Then I wanted to check the area using the formula for finding the area of a square, which is area = (length of side)^2.

So, I needed to find the length of a side. The length of one side was equal to 8 times the length of the diagonal of the right triangles formed when you cut the halves in half. Since the whole brick was 2 units by 1 unit, I knew that the halves would be 1 unit by 1 unit. When they made the triangles by cutting the small squares on the diagonals, I knew that the sides that formed the right angles of the triangle were each one unit. For right triangles, if you square the lengths of the sides that form the right angle and add them together, you get the length of the diagonal squared.

So for this triangle, 1^2 + 1^2 = (length of diagonal)^2
Which means, 1 + 1 = 2 = (length of diagonal)^2
So, the length of the diagonal = 2^1/2 (square root of 2)

So, the length of a side of the pattern sample = 8 * 2^1/2. The pattern sample is a square. So, the area of the pattern sample is the length of the side squared.
Area = (8 * 2^1/2)^2
Area = 8^2 * (2^1/2)^2
Area = 64 * 2
Area = 128 square units.

Since it matched my other solution, I knew I was right. To find the number of bricks from the area, I could take
128 square units / 2 square units per brick = 64 bricks.

Scoring Rubric

A **problem-specific rubric** can be found linked from the problem to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work.

We hope these packets are useful in helping you make the most of Pre-Algebra Problems of the Week. Please let me know if you have ideas for making them more useful.

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