NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Problem of the Week Teacher Packet

Birthday Trip

Jade's family is planning to travel to her Aunt Mazie's house to celebrate Aunt Mazie's 102nd birthday. On the first day of the trip they'll drive halfway there and then stop to set up camp for the night. On the second day of the trip they'll drive two-thirds of the remaining distance before stopping for some sightseeing and camping. On the last day they'll have 145 miles left to drive.

Question: How far is the trip to Aunt Mazie's house?

Extra: If the family averages 50 miles per hour while driving on the return trip, do you think the family will make the trip in one day? Why or why not?

Answer Check

After students submit their solution, they can choose to "check" their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually get the answer) we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

The trip is 870 miles long.

If your answer **doesn't** match ours,

- have you checked your calculations?
- if you take your answer and start at the beginning of the problem, do you get 145 miles as the distance left for the third day?
- have you reread the problem to make sure you answered the question asked?

If any of those ideas help you, you might revise your answer, and then leave a comment that tells us what you did. If you're still stuck, leave a comment that tells us where you think you need help.

If your answer **does** match ours,

- are you confident that you could solve another problem like this successfully?
- is your explanation clear and complete?
- did you make any mistakes along the way? If so, how did you find them?
- are there any hints that you would give another student?
- did you try the Extra question?

Revise your work if you have any ideas to add. Otherwise leave us a comment that tells us how you think you did—you might answer one or more of the questions above.

Our Solutions

Method 1: Work Backwards

After reading the problem I decided to use the work backwards strategy.

Since 145 was 1/3 of what they had left to drive after they drove 1/2 of the distance to Aunt Mazie's house, to

get 3/3 of half of the trip, I multiplied 145 x 3 = 435 miles, because there are 3 thirds in a whole.

Since 435 miles was only 1/2 of the trip, I multiplied 435 by 2 to get the whole trip.

435 x 2 = 870 miles

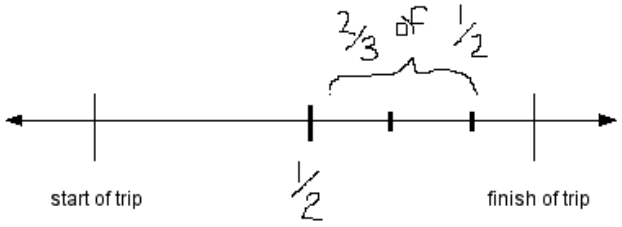
The total distance to Aunt Mazie's is 870 miles.

Extra: To start, I divided 870 miles, the distance of the trip, by 50, how many miles per hour they drove, to get the whole number without the fraction. 870 / 50 = 17 R20. They would drive 17 whole hours. The remainder was 20/50, or 2/5. 1/5 of an hour is 12 minutes, so 2/5 of an hour would be 12 min x 2 = 24 min.

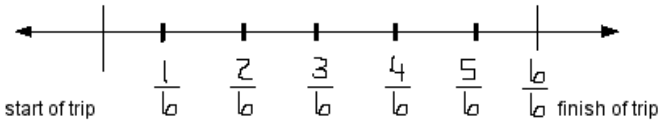
So, the whole trip would take 17 hours and 24 minutes, or almost 17 and a half hours. That would be a very long car trip to do in one day. I think they probably would do it in two days or maybe even three. But, if they had to, and had more than one driver, they could make it a one-day trip since the total travel time is less than 24 hours.

Method 2: Draw a Number Line

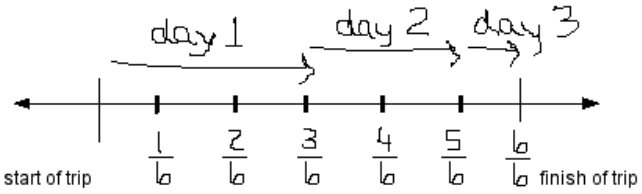
For our strategy we drew a number line showing the trip to Aunt Mazie's house. We divided the line in half, then we drew a line 2/3 of the way down the 1/2 that was left.



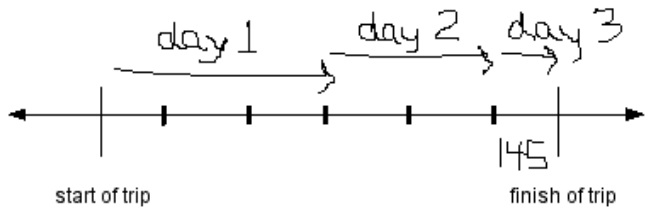
We noticed that we could divide the trip into sixths:



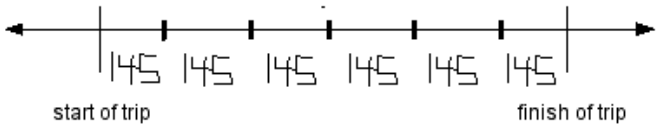
We figured out that they had traveled $\frac{3}{6}$ on day 1, $\frac{2}{6}$ on day 2, and $\frac{1}{6}$ on day 3.



We also knew from the information in the problem that day 3 was 145 miles.



This meant that $\frac{1}{6}$ = 145 miles. We could label each sixth as 145 miles:



All we need to do is add:

145 + 145 + 145 + 145 + 145 + 145

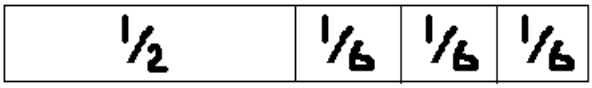
The trip is 870 miles.

Method 3: Use a Fraction Bar

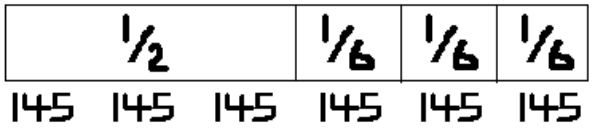
I need to find out how far is the trip to Aunt Mazie's house. I made a picture of a fraction bar to think about it. I cut the rectangle in half since they drove half way on the first day.

Then I cut the remainder into thirds since they drove two-thirds of the remainder on the second day.

A third of a half is one sixth of the whole.



I also know that the remaining sixth is 145 miles.



I multiplied 145 times six and that equaled 870 miles—the answer! That's how long the trip is to Aunt Mazie's house.

Extra: I need to find out if the family drives 50 mph how long will it take for them to get home. First I divided 870 miles by 50 miles per hour and that equaled 17.4 hours. The 17 is the amount of hours it will take them and the .4 is 4/10 of an hour. 4/10 is the same as 2/5 so: 60 X 2/5 = 24 minutes

So, yes, they could make the trip in one day. The first 5 hours one parent could drive and the next 5 hours the other parent could drive. Then take a rest and after that one of the parents could drive the 7 hours and 24 min that's left or they could each drive half of the remaining distance.

Method 4: Use Manipulatives

We read the problem and noticed:

- On Day 1 they traveled 1/2 the distance
- On Day 2 they traveled 2/3 of the remaining 1/2

- On Day 3 they traveled 145 miles

We looked in our manipulative kit on our table and got out our fraction tiles. We set these on our table:

We used the information that we noticed and made a model to show how much they traveled Day 1 and Day 2. Putting the pink 1/2 fraction tile was easy. Next we had to think about what three pieces would fit in the remaining half space. At first we were going to put the orange thirds but we realized they were too big. Three blue 1/6 tiles fit perfectly but since the problem said that only 2/3 of the distance had been traveled we knew we had to take one of them away.

That empty spot was 145 miles and since we also know the blue 1/6 tile fit in there, we realized that 1/6 of the trip equaled 145 miles. To find the full trip we multiplied 145 times 6. The trip to Aunt Mazie's house is 870 miles.

Method 5: Algebra

To solve this problem, I used an equation letting d = the total distance of the trip:

$$\frac{1}{2}d + \frac{2}{3}\left(\frac{1}{2}d\right) + 145 = d$$

First, I divided the total distance (d) by 1/2. That is how far they drove the first day. The remaining half is also d/2, so I multiplied that by 2/3 to find out far they drove the second day and added that to the distance they drove the first day. Finally, I added on the 145 miles that they still needed to drive the third day of their trip.

This is how I solved the equation:

$$\frac{1}{2}d + \frac{2}{3}\left(\frac{1}{2}d\right) + 145 = d$$

$$\frac{1}{2}d + \frac{1}{3}d + 145 = d$$

$$\frac{3}{6}d + \frac{2}{6}d + 145 = d$$

$$\frac{5}{6}d + 145 = d$$

$$145 = d - \frac{5}{6}d$$

$$145 = \frac{1}{6}d$$

$$d = 6(145)$$

$$d = 870 \text{ miles}$$

Extra: To find if the family could make the return trip in one day traveling at 50 miles per hour, you have to divide the 875 miles by the 50 miles per hour:

875 ÷ 50 = 17.4 hours

When I saw that the 17.4 hours was less than 1 day (24 hours), I realized that they could make it back in time.

Standards

If your state has adopted the Common Core State Standards, you might find the following alignments helpful.

Grade 3: Number & Operations–Fractions

Develop understanding of fractions as numbers.

Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.

Grade 4: Number & Operations–Fractions

Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

Grade 5: Number & Operations–Fractions

Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.

Teaching Suggestions

When we first offered this problem many submitters were successful in solving it and there was a nice variety of methods that were used, including working backwards, drawing pictures, and in a few cases an algebraic approach.

Most of the students who attempted the Extra found that it would take 17 R 20 or 17 2/5 or 17.4 hours to cover the distance to Aunt Mazie's house. Some submitters decided this was possible to do in one day because it is less than 24 hours. Others thought it was too much time to spend in the car. Some students referred to their own experience with car trips. It's an opportunity for a reflective comment!

Most of the common mistakes we saw were centered around communicating ideas properly. For instance, several submitters talked about the distance on the second day being 2/3 the total distance instead of 2/3 of what remained. Other submitters used the equals sign incorrectly.

Resist the urge to give direct instructions on a specific approach. Ask students to paraphrase the problem to check on their understanding before they begin working on it. Using the Noticing/Wondering activity might help students notice what is happening in this problem. Encourage them to actually make a list of those noticings. Writing them down is an important part of the process. Ask questions that help them understand the language of the problem, visualize it, and discover patterns. Good questions help students clarify their thinking and give you useful information as well.

The questions in the Answer Check, above, might serve as good prompts to help students make progress. Encourage students to use a strategy that works for them. You can see from the various methods that we have thought to use for this problem that there are several ways to approach this problem. And keep in mind that we may not have thought of them all!

I also encourage you to explore activities in the Use Logical Reasoning strategies in our Activity Series. Something there may provide the impetus that your students need.

The Online Resources Page for this problem contains links to related problems in the Problem Library and to other web-based resources.

Sample Student Solutions – Focus on *Clarity*

In the solutions below, I've provided scores the students would have received in the **Clarity** category of our scoring rubric. My comments focus on areas in which they seem to need the most improvement.

Novice	Apprentice	Practitioner	Expert
Explanation is very difficult to read and follow.	Another student might have trouble following the explanation. Long and written in one paragraph. Many spelling errors/typos.	Explains the steps that they *do* explain in such a way that another student would understand (needn't be complete to be clear). Makes an effort to check formatting, spelling, and typing (a few errors are okay).	Format and organization make ideas exceptionally clear. Answer is very readable and appealing.

David, age 12, Novice

The answer to this problem is ABOUT 290 miles.

145 + 72.5 = 217.5
217.5 + 2/3(72.5)= 290 miles

I notice that David has used several numbers that I'm not sure why he used. I also notice that he uses the word "about" in his answer and yet in his calculations he has an equals sign.

I wonder if drawing a diagram might help him think about the different days of the trip.

Adam, age 10, Novice

The trip took them 870 miles one way.

First I multiplyed 145 x 2 which equals 290, this is 1/2 way there.
Then I added 145 to 290 which is 2/3's of the way. 435 + 290 = 870.

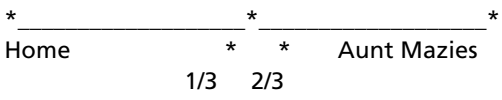
I notice that Adam has correctly stated the answer but his calculations do not result in that answer.

I might point out to him that when I add 435 and 290, I get a different answer. I would ask him to explain how he got 870.

I also might ask him why he thought that 290 miles is half-way to Aunt Mazie's house since the miles that he refers to in his first line actually represents the distance travelled during the second day.

Willie, age 12, Apprentice

145+ 2/3 + 1/2=
I know that there was 145 miles left on the last day and that they had traveled



Willie has created a helpful ASCII diagram to illustrate the trip. I notice that he mentions the 145 miles that are left to travel on the last day.

I would refer him to his diagram indicating that the 145 miles is the distance between the dot labeled 2/3 and the dot at Aunt Mazie's. I wonder if he adds in that 145 miles if he'll realize that he can figure out the lengths of the other parts.

Rachel, age 8, Apprentice

The answer is 870 mi.

I got my answer by figuring out that one sixth of the way is 145 and that times 3 is 435.435 + 435=870.

Rachel has a good start on her explanation and it would improve both her Completeness and Clarity scores if she added more details.

I might ask her if she thinks another student in her class would know from her answer how she got 1/6. I might also ask why she multiplied by 3 and why did she add 435 to 435.

A.K., age 10, Practitioner

Jade's family would go 870 miles to get to Aunt Mazie's house. On the extra the answer is yes.

1.I drew a line because it would help me to see where she stoped and where she started. I then drew a circle in the middle of the line because it says that Jade and her family stoped half way there.

2.Then I drew a circle about two-thirds from the half way point because it says that on the next day they drove two-thirds of the remaining distance.

3.Next I did 145x3 because it says that they have to go a 145 miles the next day. Also because if they went two-thirds from the half way point yesterday the 145 miles would be one-third. If you put three thirds together you will get three-thirds. So, 145x3=435.

4.That means that 435 miles is half way, so if you add two halves it would make one whole. So, 435+435=870.
870 miles is your answer.

Extra

1.She was going 50 miles an hour so, she would go 50 miles. Since it took her 870 miles to get there it would take 870 miles to get back.

2.The question is will they make it in one day. In one day there are 24 hours so, there also asking will they make it in 24 hours.

3. The answer is yes because you travel 50 miles every hour so, after 17 hours you have traveled 850 miles. Another 30 minutes and you have traveled 870 miles. 17 hours and 30 minutes is less than 24 hours.
So, the answer is yes

I notice that A. K. makes sure to explain each step and provide a reason for the action.

This is a good model to share with students who are not used to communicating in mathematics. If they remember to include "because" in their statements it might help!

D'Laney, age 13, Practitioner

The family drove a total of 870 miles to their Aunt's house. On the return trip they drove a total of 17 hours and 20 minutes. They were able to make it in one day if they average 50 miles per hour

1) On the first day they drove 1/2 of the trip.
2) On the second day they drove 2/3 of 1/2.
3) On the last day they drove the remaining distance, which equaled 145 miles.
4) To make this problem easier, I made all the fractions have common denominator.
5) I then rephrased the problem so that it sounded like this...
On the first day they drove 3/6 of the trip. On the second day they drove 2/6 of the trip. On the last day they drove 1/6 of the trip.
6) We know that 1/6 of the trip is 145 miles, so all we need to do is multiply 145 by 6. When we do that we will have 6/6 or one whole.
7) 145*6= 870
Extra
1) We know that they drove a total of 870 miles.
2) All we have to do is divide 870 by 50, because they drive at an average of 50 miles per hour.
3) 870/50= 17.20
4) That means that they could make the return trip in 17 hours and 20 minutes.
5) The final answer is YES, they could make the return trip in under one day.

I notice that D'Laney changed all of the fractions to a common denominator. I think this strategy could be one that would help other students understand better what's going on in the problem.

I like how she's numbered her steps to make it clear what she was thinking as she worked through the information given in the problem.

Kirchner's Math Whizzes, age 10, Expert

We decided to use work backwards because the first number we needed to find the answer was given at the end of the problem.

To work backwards, we figured out that 145 was one third of the distance left after the first day. We knew this because they drove 2/3 of the remaining distance on the second day, so that would mean that 145 is 1/3 of that distance because 2/3+1/3= a whole.

We found that if 145 is 1/3 of the whole, if we take it times 2, we will get 2/3, which is the distance driven on the second day. 145X2=290. If we take 145 times 3, we will know what the whole distance was for days 2 and 3. 145X3=435 miles.

Then we looked at the information for the first day. They drove halfway on the first day, so that means that 435 is equal to 1/2 of the way there. so we took 435 x 2= 870. That means that 870 miles is the whole distance to Aunt Mazie's house.

Next we checked back. We started working from the beginning of the problem with our answer- 870. If they drove halfway on the first day, that would be 870/2= 435 miles. We then subtracted 435 from 870 and got 435. That would be the beginning distance on day 2. Then the problem said they traveled 2/3 of the remaining distance on the second day. We took 2/3 of 435 which equals 290 miles. So we subtracted 290 from 435, and they had 145 miles left for the 3rd day. This matched the information in the problem so we knew we were right!

on the extra part, took the total miles- 870 and divided it by 50 so we could see if the drive would take less than one day. 870mi./50mi.per hour = 17 2/5 hours. We used equivalent fractions to find out that 2/5 of 60 minutes(1 hour)= 24 minutes. The trip would take 17 hours and 24 minutes to complete.

If there were mostly adults or older children in the car, they might have driven it in one day. If there were small children, infants, or only one adult who could drive, they would probably take more than one day to come back.

My favorite part of this team's solution is that they checked their work to make sure that everything came out as expected.

They've also done a nice job with the Extra, reflecting on the possible ages and, therefore, driving options, of the family members.

Scoring Rubric

A **problem-specific rubric** can be found linked from the problem to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work.

We hope these packets are useful in helping you make the most of the Math Fundamentals Problems of the Week. Please let me know if you have ideas for making them more useful.

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